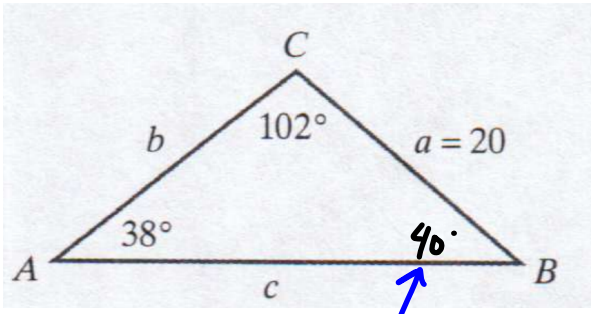
Geometry: 9.7 Part 2 The Law of Cosines

Refresher: Write a statement about when you can use Law of Sines given an oblique triangle.

This is important because when you solve triangles on your own, you will need to determine which method to use and you won't be told.

we need to know an an angle and its opposite side (same letter A & a for Example)

Ex: Solve the triangle



Since we were given A & a, we can use the Law of Sines

38 + 102 = 140

180 − 140 = 40°

$\sin 40 \cdot \frac{20}{\sin 38^\circ} = \frac{b}{\sin 40^\circ} \cdot \sin 40^\circ$

$\frac{20 \sin 40^\circ}{\sin 38^\circ} = b$

$b \approx 20.88$

$\sin 102 \cdot \frac{20}{\sin 38^\circ} = \frac{c}{\sin 102^\circ} \cdot \sin 102^\circ$

$\frac{20 \sin 102^\circ}{\sin 38} = c$

$c \approx 37.77$

9.7 Part 2 The Law of Cosines

Sometimes you need to solve a triangle and you don't have a matching pair of information (A and a, B and b, etc...)

In these situations you can use the **Law of Cosines**.

Try not to get lost in the details. This is only as complicated as you make it. Half of learning math is believing you can learn it.

The Law of Cosines (3 equations)

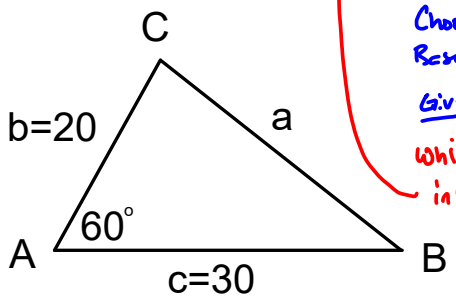
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Choose the right Equation Based on what you were given.

Given: A, b, c.

Which Equation has All that info?

Ex: Solve the triangle ABC.



$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$a^2 = 20^2 + 30^2 - 2(20)(30)\cos 60°$ ..... on calculator

$a^2 = 700$ ..... square root both sides

$\sqrt{a^2} = \sqrt{700}$

$a = \sqrt{700}$

$a \approx 26.458$

Now that we know a matching pair, we can use the Law of Sines.

Find B ← Find the Angle that is opposite the smaller side.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{20\sin 60°}{26.458} = \frac{\sin B}{20} \cdot 20$$

$$\frac{20\sin 60}{26.458} = \sin B$$

$$\sin^{-1}\left(\frac{20\sin 60}{26.458}\right) = B$$

$B \approx 40.89°$

Finding C.

$A + B + C = 180°$

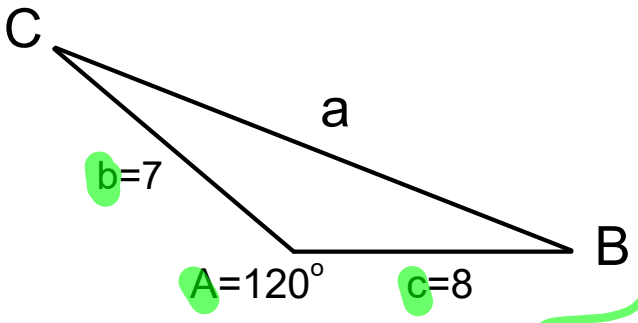
$60° + 40.89 = 100.89$

$180 - 100.89 = 79.11°$

$C = 79.11°$

9.7 Part 2 The Law of Cosines

Solve the triangle ABC.



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Use the equation that has all your given information in it.

Plug all the info in.

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 7^2 + 8^2 - 2(7)(8)\cos(120^\circ)$ on calculator

$\sqrt{a^2} = \sqrt{169}$

$a = 13$

Now find Angle B or C using the Law of Sines.

Find B.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ so $\frac{\sin B}{7} = \frac{\sin 120^\circ}{13} \cdot 7$

$\sin B = \frac{7\sin 120^\circ}{13}$ so... $\sin^{-1}\left(\frac{7\sin 120^\circ}{13}\right) = B$

$B \approx 27.8^\circ$

Finding Angle C.

Interior Angle theorem.

$\angle A + \angle B + \angle C = 180^\circ$

$120^\circ + 27.8^\circ + \angle C = 180^\circ$

$147.8 + \angle C = 180^\circ$

$-147.8 \quad -147.8$

$C = 32.2^\circ$

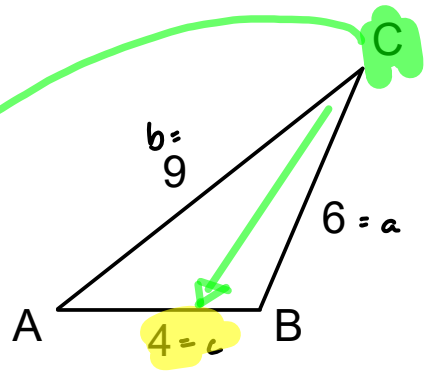
9.7 Part 2 The Law of Cosines

Ex: Solve the triangle.

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$



Strategy: Find the angle opposite the smallest side first.

$c^2 = a^2 + b^2 - 2ab\cos C$ ← Plug in what we have.

$4^2 = 6^2 + 9^2 - 2(6)(9)\cos C$

$16 = 36 + 81 - 108\cos C$

$16 = 117 - 108\cos C$

$-117 \quad -117$

$-101 = -108\cos C$

$-108 \quad -108$

$.935185 = \cos C$ inverses

$\cos^{-1}(.935185) = C$

$C \approx 20.7°$

Side Example

$4 = 6 - 2x$

$-6 \quad -6$

$-2 = -2x$

$-2 \quad -2$

Solve by the same method.

Now that we have a matching pair C & c. Use the law of Sines.

Find A.

$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow 4 \cdot \frac{\sin A}{4} = \frac{\sin 20.7°}{4} \cdot 4$

$\sin A = \frac{4\sin 20.7°}{4}$ inverse!

$\sin^{-1}\left(\frac{4\sin 20.7}{4}\right) = A$

$A = 32°$

Find B

$180° - 32 - 20.7 = B$

$B = 127.3°$

Homework:

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