Geometry: 9.7 Part 1

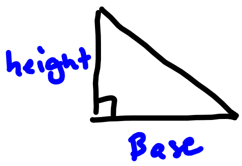
- Vocab: Oblique Triangles
- Find Area of Oblique Triangles
- Solve Oblique Triangles using the Law of Sines.

Oblique Triangle: Any Triangle that's NOT a Right Triangle.

Differences when finding area and solving oblique triangles vs right triangles:

Area: $\frac{1}{2} b \cdot h$

Right Δ

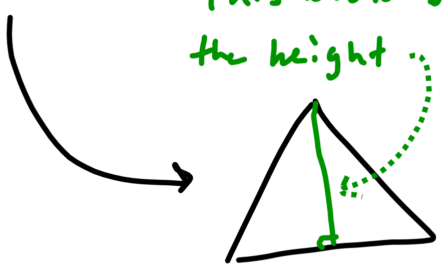


height is a side length. making it easy to use the formula.

Oblique Δ

There is no side that represents the height.

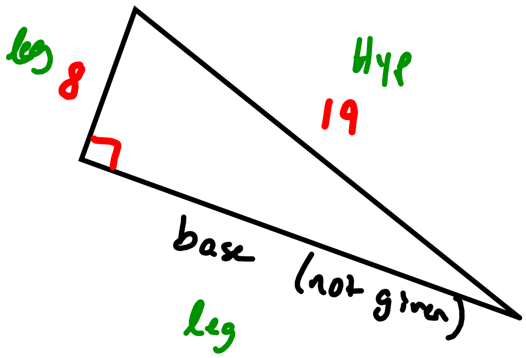
This would be the height



So Area is Harder to Find.

SOH CAH TOA Doesn't work!!

Area of Right Triangles.



Formula:

$$A = \frac{1}{2}bh$$

Find Base by Pythagorean Theorem.

$$8^2 + b^2 = 19^2$$
$$64 + b^2 = 361$$
$$-64 \qquad -64$$
$$b^2 = 297$$
$$\sqrt{b^2} = \sqrt{297}$$
$$b = \sqrt{297}$$

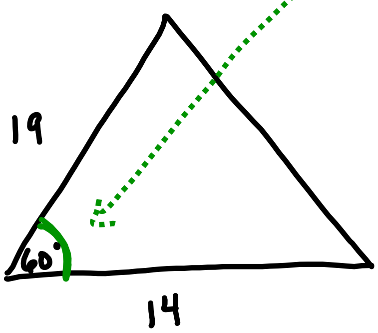
This formula is convenient because all right triangles have a side length that represents the "height" of the triangle.

$$\text{Area} = \frac{1}{2} \cdot \sqrt{297} \cdot 8 \approx 68.9 \text{ units}^2$$

Oblique Triangle Area: There is a separate formula for finding areas of oblique triangles.

$$\text{Area} = \frac{1}{2} \cdot \underset{\text{one}}{\text{side}} \cdot \underset{\text{two}}{\text{side}} \cdot \sin\left(\underset{\text{angle}}{\text{included}}\right)$$

Example:

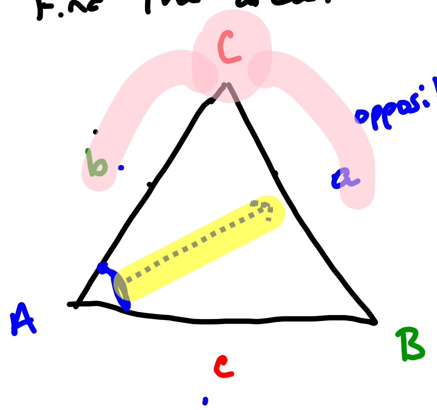


The included Angle is the one between the given sides.

$$\text{Area} = \frac{1}{2} \cdot 14 \cdot 19 \cdot \sin 60^\circ$$

$$\approx 115.1 \text{ units}^2$$

Ex: Find The area.



opposite from A.

Naming:

- Angles are capital Letters
- The sides opposite those Angles are the Lower case of the same Letter.

Ex: Given

$\angle C = 29^\circ$

$a = 38$

$b = 31$

Find Area.

$A = \frac{1}{2} \cdot 38 \cdot 31 \cdot \sin 29^\circ$

≈ 285.6 units^2

Notice $\angle C$ is the included Angle.

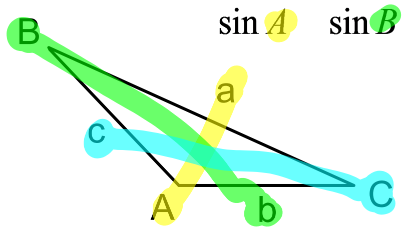
9.7 Part 1 Area and Law of Sines

The Law of Sines:

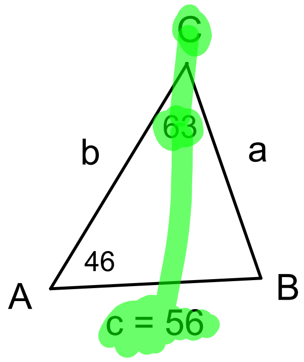
Given the following oblique triangle, the Law of Sines is... $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

or

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$



Ex: Solve the triangle.



angle B = 71°

side b = 59.4

side a = 45.2

1st: Finding B.
it is easiest.
Interior Angles Theorem
All Angles add to 180°

$63 + 46 = 109$

$180 - 109 = 71°$

you must know a matching pair (C & c For Example) to use the Law of Sines.

Put lower case letters on top if you want to find a lower case letter.

Finding side b.

$\dfrac{c}{\sin C} = \dfrac{b}{\sin B}$

Now plug in what we know.

$\sin 71° \cdot \dfrac{56}{\sin 63°} = \dfrac{b}{\cancel{\sin 71°}} \cdot \cancel{\sin 71°}$

$\dfrac{56 \cdot \sin 71}{\sin 63} = b$

on calculator

$b \approx 59.4$

Find a.

the setup $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

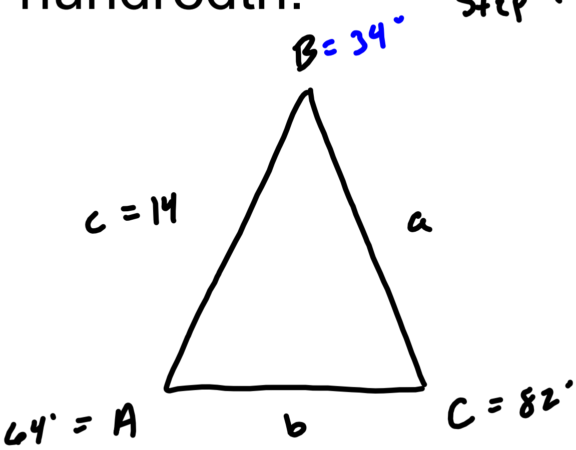
$\sin 46 \cdot \dfrac{56}{\sin 63°} = \dfrac{a}{\cancel{\sin 46°}} \cdot \cancel{\sin 46}$

$\dfrac{56 \cdot \sin 46}{\sin 63°} = a$

$a \approx 45.2$

Ex: Solve the triangle with A = 64°, C = 82°, and c = 14 centimeters. Round to the nearest hundredth.

Step 1: Draw Figure



Finding B: Interior Angles Theorem

$64° + 82° + B = 180°$

$146 + B = 180$

$B = 34°$

Finding a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\sin 64 \cdot \frac{a}{\sin 64} = \frac{14}{\sin 82} \cdot \sin 64$$

$$a = \frac{14 \sin 64}{\sin 82} \approx \boxed{12.7}$$

Finding b.

$$\sin 34 \cdot \frac{b}{\sin 34} = \frac{14}{\sin 82} \cdot \sin 34$$

$$b = \frac{14 \sin 34}{\sin 82} \approx \boxed{7.9}$$

Homework:

Page 463:

9-18