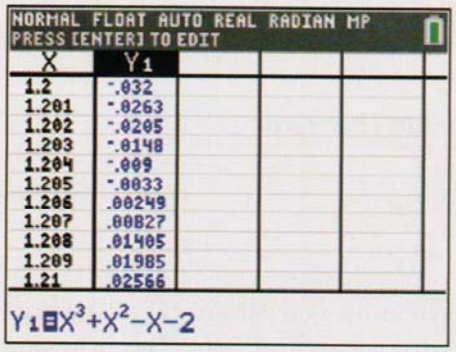


Figure 37

Now subdivide the interval [1.20, 1.21] into 10 subintervals, each of length 0.001. See Figure 37.

We conclude that the zero of the function f is 1.205, correct to three decimal places. ■

Notice that a benefit of the method used in Example 10 is that each additional iteration results in one additional decimal place of accuracy for the approximation.

NOW WORK Problem 65.

1.3 Assess Your Understanding

Concepts and Vocabulary

1. *True or False* A polynomial function is continuous at every real number.
2. *True or False* Piecewise-defined functions are never continuous at numbers where the function changes equations.
3. The three conditions necessary for a function f to be continuous at a number c are ________, ________, and ________.
4. *True or False* If f is continuous at 0, then $g(x) = \frac{1}{4}f(x)$ is continuous at 0.
5. *True or False* If f is a function defined everywhere in an open interval containing c, except possibly at c, then the number c is called a removable discontinuity of f if the function f is not continuous at c.
6. *True or False* If a function f is discontinuous at a number c, then $\lim_{x\to c} f(x)$ does not exist.
7. *True or False* If a function f is continuous on an open interval (a, b), then it is continuous on the closed interval $[a, b]$.
8. *True or False* If a function f is continuous on the closed interval $[a, b]$, then f is continuous on the open interval (a, b).

In Problems 9 and 10, explain whether each function is continuous or discontinuous on its domain.

9. The velocity of a ball thrown up into the air as a function of time, if the ball lands 5 seconds after it is thrown and stops.
10. The temperature of an oven used to bake a potato as a function of time.
11. *True or False* If a function f is continuous on a closed interval $[a, b]$, then the Intermediate Value Theorem guarantees that the function takes on every value between $f(a)$ and $f(b)$.
12. *True or False* If a function f is continuous on a closed interval $[a, b]$ and $f(a) \neq f(b)$, but both $f(a) > 0$ and $f(b) > 0$, then according to the Intermediate Value Theorem, f does not have a zero on the open interval (a, b).

Skill Building

In Problems 13–18, use the graph of $y = f(x)$ (top right).

(a) Determine if f is continuous at c.

(b) If f is discontinuous at c, state which condition(s) of the definition of continuity is (are) not satisfied.

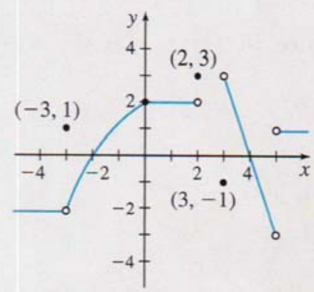
(c) If f is discontinuous at c, determine if the discontinuity is removable.

(d) If the discontinuity is removable, define (or redefine) f at c to make f continuous at c.

PAGE 105 **13.** $c = -3$ **14.** $c = 0$

15. $c = 2$ **16.** $c = 3$

17. $c = 4$ **18.** $c = 5$



In Problems 19–32, determine whether the function f is continuous at c.

PAGE 104 **19.** $f(x) = x^2 + 1$ at $c = -1$ **20.** $f(x) = x^3 - 5$ at $c = 5$

PAGE 104 **21.** $f(x) = \dfrac{x}{x^2 + 4}$ at $c = -2$ **22.** $f(x) = \dfrac{x}{x - 2}$ at $c = 2$

23. $f(x) = \begin{cases} 2x + 5 & \text{if } x \le 2 \\ 4x + 1 & \text{if } x > 2 \end{cases}$ at $c = 2$

24. $f(x) = \begin{cases} 2x + 1 & \text{if } x \le 0 \\ 2x & \text{if } x > 0 \end{cases}$ at $c = 0$

PAGE 105 **25.** $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$ at $c = 1$

26. $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$ at $c = 1$

27. $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 2x & \text{if } x > 1 \end{cases}$ at $c = 1$

28. $f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 3x & \text{if } x > 1 \end{cases}$ at $c = 1$

29. $f(x) = \begin{cases} x^2 & \text{if } x \le 0 \\ 2x & \text{if } x > 0 \end{cases}$ at $c = 0$

30. $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ 2 & \text{if } x = -1 \\ -3x + 2 & \text{if } x > -1 \end{cases}$ at $c = -1$

31. $f(x) = \begin{cases} 4 - 3x^2 & \text{if } x < 0 \\ 4 & \text{if } x = 0 \\ \sqrt{\dfrac{16 - x^2}{4 - x}} & \text{if } 0 < x < 4 \end{cases}$ at $c = 0$

32. $f(x) = \begin{cases} \sqrt{4 + x} & \text{if } -4 \le x \le 4 \\ \sqrt{\dfrac{x^2 - 3x - 4}{x - 4}} & \text{if } x > 4 \end{cases}$ at $c = 4$

In Problems 33–36, each function f has a removable discontinuity at c. Define f(c) so that f is continuous at c.

33. $f(x) = \dfrac{x^2 - 4}{x - 2}$, $c = 2$

34. $f(x) = \dfrac{x^2 + x - 12}{x - 3}$, $c = 3$

PAGE 105 **35.** $f(x) = \begin{cases} 1 + x & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$ $c = 1$

36. $f(x) = \begin{cases} x^2 + 5x & \text{if } x < -1 \\ 0 & \text{if } x = -1 \\ x - 3 & \text{if } x > -1 \end{cases}$ $c = -1$

In Problems 37–40, determine if each function f is continuous on the given interval. If the answer is no, state the interval, if any, on which f is continuous.

PAGE 107 **37.** $f(x) = \dfrac{x^2 - 9}{x - 3}$ on the interval $[-3, 3)$

38. $f(x) = 1 + \dfrac{1}{x}$ on the interval $[-1, 0)$

39. $f(x) = \dfrac{1}{\sqrt{x^2 - 9}}$ on the interval $[-3, 3]$

40. $f(x) = \sqrt{9 - x^2}$ on the interval $[-3, 3]$

In Problems 41–50, determine where each function f is continuous. First determine the domain of the function. Then support your decision using properties of continuity.

41. $f(x) = 2x^2 + 5x - \dfrac{1}{x}$

42. $f(x) = x + 1 + \dfrac{2x}{x^2 + 5}$

43. $f(x) = (x - 1)(x^2 + x + 1)$

44. $f(x) = \sqrt{x}(x^3 - 5)$

PAGE 109 **45.** $f(x) = \dfrac{x - 9}{\sqrt{x} - 3}$

46. $f(x) = \dfrac{x - 4}{\sqrt{x} - 2}$

PAGE 109 **47.** $f(x) = \sqrt{\dfrac{x^2 + 1}{2 - x}}$

48. $f(x) = \sqrt{\dfrac{4}{x^2 - 1}}$

49. $f(x) = (2x^2 + 5x - 3)^{2/3}$

50. $f(x) = (x + 2)^{1/2}$

In Problems 51–56, use the function

$$f(x) = \begin{cases} \sqrt{15 - 3x} & \text{if } x < 2 \\ \sqrt{5} & \text{if } x = 2 \\ 9 - x^2 & \text{if } 2 < x < 3 \\ \lfloor x - 2 \rfloor & \text{if } 3 \le x \end{cases}$$

51. Is f continuous at 0? Why or why not?

52. Is f continuous at 4? Why or why not?

PAGE 105 **53.** Is f continuous at 3? Why or why not?

54. Is f continuous at 2? Why or why not?

55. Is f continuous at 1? Why or why not?

56. Is f continuous at 2.5? Why or why not?

In Problems 57 and 58:

(a) *Use technology to graph f using a suitable scale on each axis.*

(b) *Based on the graph from (a), determine where f is continuous.*

(c) *Use the definition of continuity to determine where f is continuous.*

(d) *What advice would you give a fellow student about using technology to determine where a function is continuous?*

57. $f(x) = \dfrac{x^3 - 8}{x - 2}$

58. $f(x) = \dfrac{x^2 - 3x + 2}{3x - 6}$

In Problems 59–64, use the Intermediate Value Theorem to determine which of the functions must have zeros in the given interval. Indicate those for which the theorem gives no information. Do not attempt to locate the zeros.

PAGE 111 **59.** $f(x) = x^3 - 3x$ on $[-2, 2]$

60. $f(x) = x^4 - 1$ on $[-2, 2]$

61. $f(x) = \dfrac{x}{(x + 1)^2} - 1$ on $[10, 20]$

62. $f(x) = x^3 - 2x^2 - x + 2$ on $[3, 4]$

63. $f(x) = \dfrac{x^3 - 1}{x - 1}$ on $[0, 2]$

64. $f(x) = \dfrac{x^2 + 3x + 2}{x^2 - 1}$ on $[-3, 0]$

In Problems 65–72, verify that each function has a zero in the indicated interval. Then use the Intermediate Value Theorem to approximate the zero correct to three decimal places by repeatedly subdividing the interval containing the zero into 10 subintervals.

PAGE 112 **65.** $f(x) = x^3 + 3x - 5$; interval: $[1, 2]$

66. $f(x) = x^3 - 4x + 2$; interval: $[1, 2]$

67. $f(x) = 2x^3 + 3x^2 + 4x - 1$; interval: $[0, 1]$

68. $f(x) = x^3 - x^2 - 2x + 1$; interval: $[0, 1]$

69. $f(x) = x^3 - 6x - 12$; interval: $[3, 4]$

70. $f(x) = 3x^3 + 5x - 40$; interval: $[2, 3]$

71. $f(x) = x^4 - 2x^3 + 21x - 23$; interval: $[1, 2]$

72. $f(x) = x^4 - x^3 + x - 2$; interval: $[1, 2]$

In Problems 73 and 74,

(a) *Use the Intermediate Value Theorem to show that f has a zero in the given interval.*

(b) *Use technology to find the zero rounded to three decimal places.*

73. $f(x) = \sqrt{x^2 + 4x} - 2$ in $[0, 1]$

74. $f(x) = x^3 - x + 2$ in $[-2, 0]$

Applications and Extensions

Heaviside Functions *In Problems 75 and 76, determine whether the given Heaviside function is continuous at c.*

75. $u_1(t) = \begin{cases} 0 & \text{if } t < 1 \\ 1 & \text{if } t \ge 1 \end{cases}$ $c = 1$

76. $u_3(t) = \begin{cases} 0 & \text{if } t < 3 \\ 1 & \text{if } t \ge 3 \end{cases}$ $c = 3$

In Problems 77 and 78, determine where each function is continuous. Graph each function.

77. $f(x) = \begin{cases} 1 - x^2 & \text{if } |x| \le 1 \\ x^2 - 1 & \text{if } |x| > 1 \end{cases}$

78. $f(x) = \begin{cases} \sqrt{4 - x^2} & \text{if } |x| \le 2 \\ |x| - 2 & \text{if } |x| > 2 \end{cases}$

79. First-Class Mail As of April 2016, the U.S. Postal Service charged \$0.47 postage for first-class letters weighing up to and including 1 ounce, plus a flat fee of \$0.21 for each additional or partial ounce up to 3.5 ounces. First-class letter rates do not apply to letters weighing more than 3.5 ounces.

Source: U.S. Postal Service Notice 123.

(a) Find a function C that models the first-class postage charged for a letter weighing w ounces. Assume $w > 0$.
(b) What is the domain of C?
(c) Determine the intervals on which C is continuous.
(d) At numbers where C is not continuous (if any), what type of discontinuity does C have?
(e) What are the practical implications of the answer to (d)?

80. First-Class Mail As of April 2016, the U.S. Postal Service charged \$0.94 postage for first-class large envelopes weighing up to and including 1 ounce, plus a flat fee of \$0.21 for each additional or partial ounce up to 13 ounces. First-class rates do not apply to large envelopes weighing more than 13 ounces.

Source: U.S. Postal Service Notice 123.

(a) Find a function C that models the first-class postage charged for a large envelope weighing w ounces. Assume $w > 0$.
(b) What is the domain of C?
(c) Determine the intervals on which C is continuous.
(d) At numbers where C is not continuous (if any), what type of discontinuity does C have?
(e) What are the practical implications of the answer to (d)?

81. Cost of Electricity In June 2016, Florida Power and Light had the following monthly rate schedule for electric usage in single-family residences:

Monthly customer charge	\$7.87
Fuel charge	
≤ 1000 kWH	0.02173 per kWH
> 1000 kWH	\$21.73 + 0.03173 for each kWH in excess of 1000

Source: Florida Power and Light, Miami, FL.

(a) Find a function C that models the monthly cost of using x kWH of electricity.
(b) What is the domain of C?
(c) Determine the intervals on which C is continuous.
(d) At numbers where C is not continuous (if any), what type of discontinuity does C have?
(e) What are the practical implications of the answer to (d)?

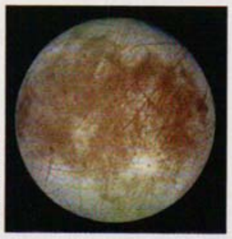
82. Cost of Water The Jericho Water District determines quarterly water costs, in dollars, using the following rate schedule:

Water used (in thousands of gallons)	Cost
$0 \le x \le 10$	\$9.00
$10 < x \le 30$	\$9.00 + 0.95 for each thousand gallons in excess of 10,000 gallons
$30 < x \le 100$	\$28.00 + 1.65 for each thousand gallons in excess of 30,000 gallons
$x > 100$	\$143.50 + 2.20 for each thousand gallons in excess of 100,000 gallons

Source: Jericho Water District, Syosset, NY.

(a) Find a function C that models the quarterly cost of using x thousand gallons of water.
(b) What is the domain of C?
(c) Determine the intervals on which C is continuous.
(d) At numbers where C is not continuous (if any), what type of discontinuity does C have?
(e) What are the practical implications of the answer to (d)?

83. Gravity on Europa Europa, one of the larger satellites of Jupiter, has an icy surface and appears to have oceans beneath the ice. This makes it a candidate for possible extraterrestrial life. Because Europa is much smaller than most planets, its gravity is weaker. If we think of Europa as a sphere with uniform internal density, then inside the sphere, the gravitational field g is given by $g(r) = \frac{Gm}{R^3}r$, $0 \le r < R$, where R is the radius of the sphere, r is the distance from the center of the sphere, and G is the universal gravitation constant. Outside a uniform sphere of mass m, the gravitational field g is given by



NASA/JPL/DLR

$$g(r) = \frac{Gm}{r^2}, \quad R < r$$

(a) For the gravitational field of Europa to be continuous at its surface, what must $g(r)$ equal? [*Hint:* Investigate $\lim_{r \to R} g(r)$.]
(b) Determine the gravitational field at Europa's surface. This will indicate the type of gravity environment organisms will experience. Use the following measured values: Europa's mass is 4.8×10^{22} kilograms, its radius is 1.569×10^6 meters, and $G = 6.67 \times 10^{-11}$.
(c) Compare the result found in (b) to the gravitational field on Earth's surface, which is 9.8 meter/second^2. Is the gravity on Europa less than or greater than that on Earth?

84. Find constants A and B so that the function below is continuous for all x. Graph the resulting function.

$$f(x) = \begin{cases} (x-1)^2 & \text{if } -\infty < x < 0 \\ (A - x)^2 & \text{if } \quad 0 \le x < 1 \\ x + B & \text{if } \quad 1 \le x < \infty \end{cases}$$

85. Find constants A and B so that the function below is continuous for all x. Graph the resulting function.

$$f(x) = \begin{cases} x + A & \text{if } -\infty < x < 4 \\ (x-1)^2 & \text{if } \quad 4 \le x \le 9 \\ Bx + 1 & \text{if } \quad 9 < x < \infty \end{cases}$$

86. For the function f below, find k so that f is continuous at 2.

$$f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & \text{if } x \geq -\dfrac{5}{2}, x \neq 2 \\ k & \text{if } x = 2 \end{cases}$$

87. Suppose $f(x) = \dfrac{x^2 - 6x - 16}{(x^2 - 7x - 8)\sqrt{x^2 - 4}}$.

(a) For what numbers x is f defined?

(b) For what numbers x is f discontinuous?

(c) Which discontinuities found in (b) are removable?

88. Intermediate Value Theorem

(a) Use the Intermediate Value Theorem to show that the function $f(x) = \sin x + x - 3$ has a zero in the interval $[0, \pi]$.

(b) Approximate the zero rounded to three decimal places.

89. Intermediate Value Theorem

(a) Use the Intermediate Value Theorem to show that the function $f(x) = e^x + x - 2$ has a zero in the interval $[0, 2]$.

(b) Approximate the zero rounded to three decimal places.

In Problems 90–93, verify that each function intersects the given line in the indicated interval. Then use the Intermediate Value Theorem to approximate the point of intersection correct to three decimal places by repeatedly subdividing the interval into 10 subintervals.

90. $f(x) = x^3 - 2x^2 - 1$; line: $y = -1$; interval: $(1, 4)$

91. $g(x) = -x^4 + 3x^2 + 3$; line: $y = 3$; interval: $(1, 2)$

92. $h(x) = \dfrac{x^3 - 5}{x^2 + 1}$; line: $y = 1$; interval: $(1, 3)$

93. $r(x) = \dfrac{x - 6}{x^2 + 2}$; line: $y = -1$; interval: $(0, 3)$

94. Graph a function that is continuous on the closed interval $[5, 12]$, that is negative at both endpoints and has exactly three distinct zeros in this interval. Does this contradict the Intermediate Value Theorem? Explain.

95. Graph a function that is continuous on the closed interval $[-1, 2]$, that is positive at both endpoints and has exactly two zeros in this interval. Does this contradict the Intermediate Value Theorem? Explain.

96. Graph a function that is continuous on the closed interval $[-2, 3]$, is positive at -2 and negative at 3 and has exactly two zeros in this interval. Is this possible? Does this contradict the Intermediate Value Theorem? Explain.

97. Graph a function that is continuous on the closed interval $[-5, 0]$, is negative at -5 and positive at 0 and has exactly three zeros in the interval. Is this possible? Does this contradict the Intermediate Value Theorem? Explain.

98. (a) Explain why the Intermediate Value Theorem gives no information about the zeros of the function $f(x) = x^4 - 1$ on the interval $[-2, 2]$.

(b) Use technology to determine whether or not f has a zero on the interval $[-2, 2]$.

99. (a) Explain why the Intermediate Value Theorem gives no information about the zeros of the function $f(x) = \ln(x^2 + 2)$ on the interval $[-2, 2]$.

(b) Use technology to determine whether or not f has a zero on the interval $[-2, 2]$.

100. Intermediate Value Theorem

(a) Use the Intermediate Value Theorem to show that the functions $y = x^3$ and $y = 1 - x^2$ intersect somewhere between $x = 0$ and $x = 1$.

(b) Use technology to find the coordinates of the point of intersection rounded to three decimal places.

(c) Use technology to graph both functions on the same set of axes. Be sure the graph shows the point of intersection.

101. Intermediate Value Theorem An airplane is travelling at a speed of 620 miles per hour and then encounters a slight headwind that slows it to 608 miles per hour. After a few minutes, the headwind eases and the plane's speed increases to 614 miles per hour. Explain why the plane's speed is 610 miles per hour on at least two different occasions during the flight.
Source: Submitted by the students of Millikin University.

102. Suppose a function f is defined and continuous on the closed interval $[a, b]$. Is the function $h(x) = \dfrac{1}{f(x)}$ also continuous on the closed interval $[a, b]$? Discuss the continuity of h on $[a, b]$.

103. Given the two functions f and h:

$$f(x) = x^3 - 3x^2 - 4x + 12 \qquad h(x) = \begin{cases} \dfrac{f(x)}{x-3} & \text{if } x \neq 3 \\ p & \text{if } x = 3 \end{cases}$$

(a) Find all the zeros of the function f.

(b) Find the number p so that the function h is continuous at $x = 3$. Justify your answer.

(c) Determine whether h, with the number found in (b), is even, odd, or neither. Justify your answer.

104. The function $f(x) = \dfrac{|x|}{x}$ is not defined at 0. Explain why it is impossible to define $f(0)$ so that f is continuous at 0.

105. Find two functions f and g that are each continuous at c, yet $\dfrac{f}{g}$ is not continuous at c.

106. Discuss the difference between a discontinuity that is removable and one that is nonremovable. Give an example of each.

Bisection Method for Approximating Zeros of a Function *Suppose the Intermediate Value Theorem indicates that a function f has a zero in the interval (a, b). The bisection method approximates the zero by evaluating f at the midpoint m_1 of the interval (a, b). If $f(m_1) = 0$, then m_1 is the zero we seek and the process ends. If $f(m_1) \neq 0$, then the sign of $f(m_1)$ is opposite that of either $f(a)$ or $f(b)$ (but not both), and the zero lies in that subinterval. Evaluate f at the midpoint m_2 of this subinterval. Continue bisecting the subinterval containing the zero until the desired degree of accuracy is obtained.*

In Problems 107–114, use the bisection method three times to approximate the zero of each function in the given interval.

107. $f(x) = x^3 + 3x - 5$; interval: $[1, 2]$

108. $f(x) = x^3 - 4x + 2$; interval: $[1, 2]$

109. $f(x) = 2x^3 + 3x^2 + 4x - 1$; interval: $[0, 1]$

110. $f(x) = x^3 - x^2 - 2x + 1$; interval: $[0, 1]$

111. $f(x) = x^3 - 6x - 12$; interval: $[3, 4]$

112. $f(x) = 3x^3 + 5x - 40$; interval: $[2, 3]$

113. $f(x) = x^4 - 2x^3 + 21x - 23$; interval $[1, 2]$

114. $f(x) = x^4 - x^3 + x - 2$; interval: $[1, 2]$

115. Intermediate Value Theorem Use the Intermediate Value Theorem to show that the function $f(x) = \sqrt{x^2 + 4x} - 2$ has a zero in the interval $[0, 1]$. Then approximate the zero correct to one decimal place.

116. Intermediate Value Theorem Use the Intermediate Value Theorem to show that the function $f(x) = x^3 - x + 2$ has a zero in the interval $[-2, 0]$. Then approximate the zero correct to two decimal places.

117. Continuity of a Sum If f and g are each continuous at c, prove that $f + g$ is continuous at c. (*Hint:* Use the Limit of a Sum Property.)

118. Intermediate Value Theorem Suppose that the functions f and g are continuous on the interval $[a, b]$. If $f(a) < g(a)$ and $f(b) > g(b)$, prove that the graphs of $y = f(x)$ and $y = g(x)$ intersect somewhere between $x = a$ and $x = b$. [*Hint:* Define $h(x) = f(x) - g(x)$ and show $h(x) = 0$ for some x between a and b.]

Challenge Problems

119. Intermediate Value Theorem Let $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x-2}$. Use the Intermediate Value Theorem to prove that there is a real number c between 1 and 2 for which $f(c) = 0$.

120. Intermediate Value Theorem Prove that there is a real number c between 2.64 and 2.65 for which $c^2 = 7$.

121. Show that the existence of $\lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$ implies f is continuous at $x = a$.

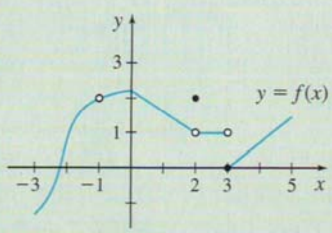
122. Find constants A, B, C, and D so that the function below is continuous for all x. Sketch the graph of the resulting function.

$$f(x) = \begin{cases} \dfrac{x^2 + x - 2}{x - 1} & \text{if } -\infty < x < 1 \\ A & \text{if } x = 1 \\ B(x - C)^2 & \text{if } 1 < x < 4 \\ D & \text{if } x = 4 \\ 2x - 8 & \text{if } 4 < x < \infty \end{cases}$$

123. Let f be a function for which $0 \le f(x) \le 1$ for all x in $[0, 1]$. If f is continuous on $[0, 1]$, show that there exists at least one number c in $[0, 1]$ such that $f(c) = c$. [*Hint:* Let $g(x) = x - f(x)$.]

AP® Practice Problems

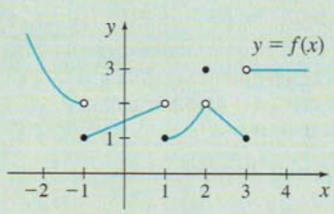
PAGE 103 **1.** The graph of a function f is shown below. Where on the open interval $(-3, 5)$ is f discontinuous?



(A) 3 only (B) -1 and 3 only

(C) 1 only (D) -1, 2, and 3

PAGE 103 **2.** The graph of a function f is shown below.



If $\lim\limits_{x\to c} f(x)$ exists and if f is not continuous at c, then $c =$

(A) -1 (B) 1 (C) 2 (D) 3

PAGE 104 **3.** If the function $f(x) = \dfrac{x^2 - 25}{x + 5}$ is continuous at -5, then $f(-5) =$

(A) -10 (B) -5 (C) 0 (D) 10

PAGE 105 **4.** If $f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+15}}{x - 10} & \text{if } x \ne 10 \\ k & \text{if } x = 10 \end{cases}$

and if f is continuous at $x = 10$, then $k =$

(A) 0 (B) $\dfrac{1}{10}$ (C) 1 (D) 10

PAGE 105 **5.** If $\lim\limits_{x\to c} f(x) = L$, where L is a real number, which of the following must be true?

(A) f is defined at $x = c$. (B) f is continuous at $x = c$.

(C) $f(c) = L$. (D) None of the above.

PAGE 111 **6.** If $f(x) = x^3 - 2x + 5$ and if $f(c) = 0$ for only one real number c, then c is between

(A) -4 and -2 (B) -2 and -1 (C) -1 and 1 (D) 1 and 3

PAGE 111 **7.** The function f is continuous at all real numbers, and $f(-8) = 3$ and $f(-1) = -4$. If f has only one real zero (root), then which number x could satisfy $f(x) = 0$?

(A) -10 (B) -5 (C) 0 (D) 2

PAGE 105 **8.** Let f be the function defined by

$$f(x) = \begin{cases} x^2 & \text{if} \quad x < 0 \\ \sqrt{x} & \text{if} \quad 0 \le x < 1 \\ 2 - x & \text{if} \quad 1 \le x < 2 \\ x - 3 & \text{if} \quad x \ge 2 \end{cases}$$

For what numbers x is f NOT continuous?

(A) 1 only (B) 2 only
(C) 0 and 2 only (D) 1 and 2 only

PAGE 111 **9.** The function f is continuous on the closed interval $[-2, 6]$. If $f(-2) = 7$ and $f(6) = -1$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$.
(B) $f(c) = 2$ for at least one number c between -2 and 6.
(C) $f(c) = 0$ for at least one number c between -1 and 7.
(D) $-1 \le f(x) \le 7$ for all numbers in the closed interval $[-2, 6]$.

PAGE 111 **10.** The function f is continuous on the closed interval $[-2, 2]$. Several values of the function f are given in the table below.

x	-2	0	2
$f(x)$	3	c	2

The equation $f(x) = 1$ must have at least two solutions in the interval $[-2, 2]$ if $c =$

(A) $\frac{1}{2}$ (B) 1 (C) 3 (D) 4

PAGE 105 **11.** The function f is defined by $f(x) = \begin{cases} x^2 - 2x + 3 & \text{if } x \le 1 \\ -2x + 5 & \text{if } x > 1 \end{cases}$

(a) Is f continuous at $x = 1$?
(b) Use the definition of continuity to explain your answer.

1.4 Limits and Continuity of Trigonometric, Exponential, and Logarithmic Functions

OBJECTIVES ***When you finish this section, you should be able to:***

1. **Use the Squeeze Theorem to find a limit (p. 117)**
2. **Find limits involving trigonometric functions (p. 119)**
3. **Determine where the trigonometric functions are continuous (p. 122)**
4. **Determine where an exponential or a logarithmic function is continuous (p. 124)**

Until now we have found limits using the basic limits

$$\lim_{x \to c} A = A \qquad \lim_{x \to c} x = c$$

and properties of limits. But there are many limit problems that cannot be found by directly applying these techniques. To find such limits requires different results, such as the *Squeeze Theorem**, or basic limits involving trigonometric and exponential functions.

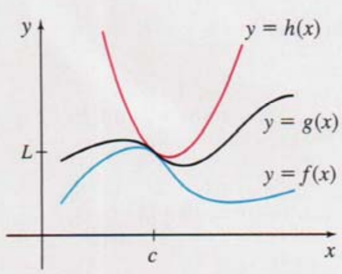
1 Use the Squeeze Theorem to Find a Limit

To use the Squeeze Theorem to find $\lim_{x \to c} g(x)$, we need to know, or be able to find, two functions f and h that "sandwich" the function g between them for all x close to c. That is, in some interval containing c, the functions f, g, and h satisfy the inequality

$$f(x) \le g(x) \le h(x)$$

Then if f and h have the same limit L as x approaches c, the function g is "squeezed" to the same limit L as x approaches c. See Figure 38.

We state the Squeeze Theorem here. The proof is given in Appendix B.



$\lim_{x \to c} f(x) = L$, $\lim_{x \to c} h(x) = L$, $\lim_{x \to c} g(x) = L$

Figure 38

*The Squeeze Theorem is also known as the Sandwich Theorem and the Pinching Theorem.