Study Tip

Much of the essential information in this chapter can be found in three places:

- Study Tip on page 524, showing special angles and how to obtain exact values of trigonometric functions at these angles
- **Table 5.6** on page 566, showing the graphs of the six trigonometric functions, with their domains, ranges, and periods
- **Table 5.10** on page 578, showing graphs of the three basic inverse trigonometric functions, with their domains and ranges.

Make copies of these pages and mount them on cardstock. Use this reference sheet as you work the review exercises until you have all the information on the reference sheet memorized for the chapter test.

Review Exercises

5.1

1. Find the radian measure of the central angle of a circle of radius 6 centimeters that intercepts an arc of length 27 centimeters.

In Exercises 2–4, convert each angle in degrees to radians. Express your answer as a multiple of π.

2. $15°$ **3.** $120°$ **4.** $315°$

In Exercises 5–7, convert each angle in radians to degrees.

5. $\frac{5\pi}{3}$ **6.** $\frac{7\pi}{5}$ **7.** $-\frac{5\pi}{6}$

In Exercises 8–12, draw each angle in standard position.

8. $\frac{5\pi}{6}$ **9.** $-\frac{2\pi}{3}$ **10.** $\frac{8\pi}{3}$

11. $190°$ **12.** $-135°$

In Exercises 13–17, find a positive angle less than 360° or 2π that is coterminal with the given angle.

13. $400°$ **14.** $-445°$ **15.** $\frac{13\pi}{4}$

16. $\frac{31\pi}{6}$ **17.** $-\frac{8\pi}{3}$

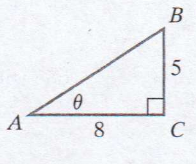
18. Find the length of the arc on a circle of radius 10 feet intercepted by a 135° central angle. Express arc length in terms of π. Then round your answer to two decimal places.

19. The angular speed of a propeller on a wind generator is 10.3 revolutions per minute. Express this angular speed in radians per minute.

20. The propeller of an airplane has a radius of 3 feet. The propeller is rotating at 2250 revolutions per minute. Find the linear speed, in feet per minute, of the tip of the propeller.

5.2

21. Use the triangle to find each of the six trigonometric functions of θ.



In Exercises 22–25, find the exact value of each expression. Do not use a calculator.

22. $\sin\frac{\pi}{6} + \tan^2\frac{\pi}{3}$ **23.** $\cos^2\frac{\pi}{4} - \tan^2\frac{\pi}{4}$

24. $\sec^2\frac{\pi}{5} - \tan^2\frac{\pi}{5}$ **25.** $\cos\frac{2\pi}{9}\sec\frac{2\pi}{9}$

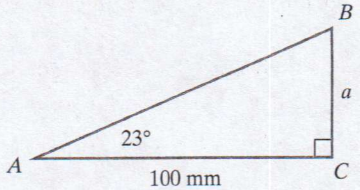
26. If θ is an acute angle and $\sin\theta = \frac{2\sqrt{7}}{7}$, use the identity $\sin^2\theta + \cos^2\theta = 1$ to find $\cos\theta$.

In Exercises 27–28, find a cofunction with the same value as the given expression.

27. $\sin 70°$ **28.** $\cos\frac{\pi}{2}$

In Exercises 29–31, find the measure of the side of the right triangle whose length is designated by a lowercase letter. Round answers to the nearest whole number.

29.



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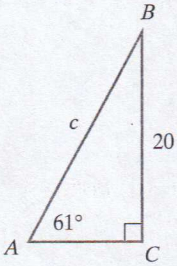

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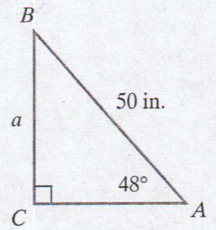
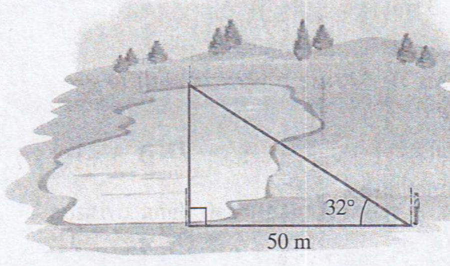


32. If $\sin\theta = \frac{1}{4}$ and θ is acute, find $\tan\left(\frac{\pi}{2} - \theta\right)$.

33. A hiker climbs for a half mile up a slope whose inclination is 17°. How many feet of altitude, to the nearest foot, does the hiker gain?

34. To find the distance across a lake, a surveyor took the measurements in the figure shown. What is the distance across the lake? Round to the nearest meter.



35. When a six-foot pole casts a four-foot shadow, what is the angle of elevation of the sun? Round to the nearest whole degree.

5.3 and 5.4

In Exercises 36–37, a point on the terminal side of angle θ is given. Find the exact value of each of the six trigonometric functions of θ, or state that the function is undefined.

36. $(-1, -5)$ **37.** $(0, -1)$

In Exercises 38–39, let θ be an angle in standard position. Name the quadrant in which θ lies.

38. $\tan\theta > 0$ and $\sec\theta > 0$ **39.** $\tan\theta > 0$ and $\cos\theta < 0$

In Exercises 40–42, find the exact value of each of the remaining trigonometric functions of θ.

40. $\cos\theta = \frac{2}{5}, \sin\theta < 0$ **41.** $\tan\theta = -\frac{1}{3}, \sin\theta > 0$

42. $\cot\theta = 3, \cos\theta < 0$

In Exercises 43–47, find the reference angle for each angle.

43. $265°$ **44.** $\frac{5\pi}{8}$ **45.** $-410°$

46. $\frac{17\pi}{6}$ **47.** $-\frac{11\pi}{3}$

In Exercises 48–58, find the exact value of each expression. Do not use a calculator.

48. $\sin 240°$ **49.** $\tan 120°$ **50.** $\sec\frac{7\pi}{4}$

51. $\cos\frac{11\pi}{6}$ **52.** $\cot(-210°)$ **53.** $\csc\left(-\frac{2\pi}{3}\right)$

54. $\sin\left(-\frac{\pi}{3}\right)$ **55.** $\sin 495°$ **56.** $\tan\frac{13\pi}{4}$

57. $\sin\frac{22\pi}{3}$ **58.** $\cos\left(-\frac{35\pi}{6}\right)$

5.5

In Exercises 59–64, determine the amplitude and period of each function. Then graph one period of the function.

59. $y = 3\sin 4x$ **60.** $y = -2\cos 2x$

61. $y = 2\cos\frac{1}{2}x$ **62.** $y = \frac{1}{2}\sin\frac{\pi}{3}x$

63. $y = -\sin\pi x$ **64.** $y = 3\cos\frac{x}{3}$

In Exercises 65–69, determine the amplitude, period, and phase shift of each function. Then graph one period of the function.

65. $y = 2\sin(x - \pi)$ **66.** $y = -3\cos(x + \pi)$

67. $y = \frac{3}{2}\cos\left(2x + \frac{\pi}{4}\right)$ **68.** $y = \frac{5}{2}\sin\left(2x + \frac{\pi}{2}\right)$

69. $y = -3\sin\left(\frac{\pi}{3}x - 3\pi\right)$

In Exercises 70–71, use a vertical shift to graph one period of the function.

70. $y = \sin 2x + 1$ **71.** $y = 2\cos\frac{1}{3}x - 2$

72. The function

$$y = 98.6 + 0.3\sin\left(\frac{\pi}{12}x - \frac{11\pi}{12}\right)$$

models variation in body temperature, y, in °F, x hours after midnight.

a. What is body temperature at midnight?

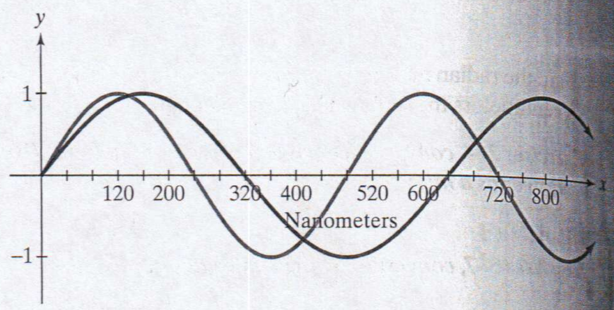
b. What is the period of the body temperature cycle?

c. When is body temperature highest? What is the body temperature at this time?

d. When is body temperature lowest? What is the body temperature at this time?

e. Graph one period of the body temperature function.

73. Light waves can be modeled by sine functions. The graphs show waves of red and blue light. Write an equation in the form $y = A\sin Bx$ that models each of these light waves.



5.6

In Exercises 74–80, graph two full periods of the given tangent or cotangent function.

74. $y = 4\tan 2x$ **75.** $y = -2\tan\frac{\pi}{4}x$

76. $y = \tan(x + \pi)$ **77.** $y = -\tan\left(x - \frac{\pi}{4}\right)$

78. $y = 2\cot 3x$ **79.** $y = -\frac{1}{2}\cot\frac{\pi}{2}x$

80. $y = 2\cot\left(x + \frac{\pi}{2}\right)$

In Exercises 81–84, graph two full periods of the given cosecant or secant function.

81. $y = 3\sec 2\pi x$ **82.** $y = -2\csc\pi x$

83. $y = 3\sec(x + \pi)$ **84.** $y = \frac{5}{2}\csc(x - \pi)$

5.7

In Exercises 85–103, find the exact value of each expression. Do not use a calculator.

85. $\sin^{-1} 1$ **86.** $\cos^{-1} 1$

87. $\tan^{-1} 1$ **88.** $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

89. $\cos^{-1}\left(-\frac{1}{2}\right)$ **90.** $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

91. $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$ **92.** $\sin(\cos^{-1} 0)$

93. $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$ **94.** $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

95. $\csc\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$ **96.** $\cos\left(\tan^{-1}\frac{3}{4}\right)$

97. $\sin\left(\cos^{-1}\frac{3}{5}\right)$

98. $\tan\left[\sin^{-1}\left(-\frac{3}{5}\right)\right]$

99. $\tan\left[\cos^{-1}\left(-\frac{4}{5}\right)\right]$

100. $\sin\left[\tan^{-1}\left(-\frac{1}{3}\right)\right]$

101. $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

102. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

103. $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$

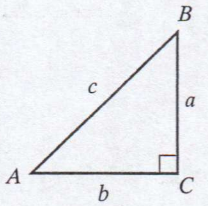
In Exercises 104–105, use a right triangle to write each expression as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x.

104. $\cos\left(\tan^{-1}\frac{x}{2}\right)$

105. $\sec\left(\sin^{-1}\frac{1}{x}\right)$

5.8

In Exercises 106–109, solve the right triangle shown in the figure. Round lengths to two decimal places and express angles to the nearest tenth of a degree.



106. $A = 22.3°, c = 10$

107. $B = 37.4°, b = 6$

108. $a = 2, c = 7$

109. $a = 1.4, b = 3.6$

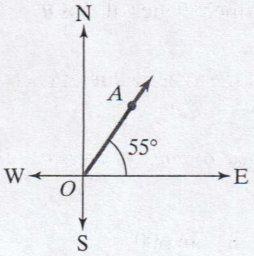
110. From a point on level ground 80 feet from the base of a building, the angle of elevation is 25.6°. Approximate the height of the building to the nearest foot.

111. Two buildings with flat roofs are 60 yards apart. The height of the shorter building is 40 yards. From its roof, the angle of elevation to the edge of the roof of the taller building is 40°. Find the height of the taller building to the nearest yard.

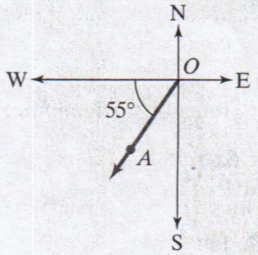
112. You want to measure the height of an antenna on the top of a 125-foot building. From a point in front of the building, you measure the angle of elevation to the top of the building to be 68° and the angle of elevation to the top of the antenna to be 71°. How tall is the antenna, to the nearest tenth of a foot?

In Exercises 113–114, use the figures shown to find the bearing from O to A.

113.



114.



115. A ship is due west of a lighthouse. A second ship is 12 miles south of the first ship. The bearing from the second ship to the lighthouse is N 64° E. How far, to the nearest tenth of a mile, is the first ship from the lighthouse?

116. From city A to city B, a plane flies 850 miles at a bearing of N 58° E. From city B to city C, the plane flies 960 miles at a bearing of S 32° E.

a. Find, to the nearest tenth of a mile, the distance from city A to city C.

b. What is the bearing from city A to city C?

In Exercises 117–118, an object moves in simple harmonic motion described by the given equation, where t is measured in seconds and d in centimeters. In each exercise, find:

a. the maximum displacement

b. the frequency

c. the time required for one cycle.

117. $d = 20\cos\frac{\pi}{4}t$

118. $d = \frac{1}{2}\sin 4t$

In Exercises 119–120, an object is attached to a coiled spring. In Exercise 119, the object is pulled down (negative direction from the rest position) and then released. In Exercise 120, the object is initially at its rest position. After that, it is pulled down and then released. Write an equation for the distance of the object from its rest position after t seconds.

Distance from Rest Position at $t = 0$	Amplitude	Period
119. 30 inches	30 inches	2 seconds
120. 0 inches	$\frac{1}{4}$ inch	5 seconds

CHAPTER Test Prep VIDEOS

Chapter 5 Test

1. Convert 135° to an exact radian measure.

2. Find the length of the arc on a circle of radius 20 feet intercepted by a 75° central angle. Express arc length in terms of π. Then round your answer to two decimal places.

3. **a.** Find a positive angle less than 2π that is coterminal with $\frac{16\pi}{3}$.

b. Find the reference angle for $\frac{16\pi}{3}$.

DEFINITIONS AND CONCEPTS	EXAMPLES
g. Vector $\mathbf{v}$, from $(0, 0)$ to (a, b), called a position vector, is represented as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$, where a is the horizontal component and b is the vertical component. The magnitude of $\mathbf{v}$ is given by $\|\mathbf{v}\| = \sqrt{a^2 + b^2}$.	Ex. 2, p. 722
h. Vector $\mathbf{v}$ from (x_1, y_1) to (x_2, y_2) is equal to the position vector $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$. In rectangular coordinates, the term "vector" refers to the position vector in terms of $\mathbf{i}$ and $\mathbf{j}$ that is equal to it.	Ex. 3, p. 723
i. Operations with Vectors in Terms of $\mathbf{i}$ and $\mathbf{j}$ If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, then • $\mathbf{v} + \mathbf{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j}$ • $\mathbf{v} - \mathbf{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j}$ • $k\mathbf{v} = (ka_1)\mathbf{i} + (kb_1)\mathbf{j}$	Ex. 4, p. 724; Ex. 5, p. 724; Ex. 6, p. 725
j. The zero vector $\mathbf{0}$ is the vector whose magnitude is 0 and is assigned no direction. Many properties of vector addition and scalar multiplication involve the zero vector. Some of these properties are listed in the box on page 725.	
k. The vector $\frac{\mathbf{v}}{\|\mathbf{v}\|}$ is the unit vector that has the same direction as $\mathbf{v}$.	Ex. 7, p. 726
l. A vector with magnitude $\|\mathbf{v}\|$ and direction angle θ, the angle that $\mathbf{v}$ makes with the positive x-axis, can be expressed in terms of its magnitude and direction angle as $\mathbf{v} = \|\mathbf{v}\| \cos\theta\mathbf{i} + \|\mathbf{v}\| \sin\theta\mathbf{j}$.	Ex. 8, p. 727; Ex. 9, p. 727
7.7 The Dot Product	
a. Definition of the Dot Product If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, the dot product of $\mathbf{v}$ and $\mathbf{w}$ is defined by $\mathbf{v} \cdot \mathbf{w} = a_1a_2 + b_1b_2$.	Ex. 1, p. 733
b. Alternative Formula for the Dot Product: $\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\| \cos\theta$, where θ is the smallest nonnegative angle between $\mathbf{v}$ and $\mathbf{w}$	
c. Angle between Two Vectors $\cos\theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$ and $\theta = \cos^{-1}\left(\frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}\right)$	Ex. 2, p. 735
d. Two vectors are orthogonal when the angle between them is 90°. To show that two vectors are orthogonal, show that their dot product is zero.	Ex. 3, p. 736
e. The vector projection of $\mathbf{v}$ onto $\mathbf{w}$ is given by $\text{proj}_{\mathbf{w}}\mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$.	Ex. 4, p. 737
f. Expressing a vector as the sum of two orthogonal vectors, called the vector components, is shown in the box on page 737.	Ex. 5, p. 738
g. The work, W, done by a force $\mathbf{F}$ moving an object from A to B is $W = \mathbf{F} \cdot \overrightarrow{AB}$. Thus, $W = \|\mathbf{F}\|\|\overrightarrow{AB}\| \cos\theta$, where θ is the angle between the force and the direction of motion.	Ex. 6, p. 739

Review Exercises

7.1 and 7.2

In Exercises 1–12, solve each triangle. Round lengths to the nearest tenth and angle measures to the nearest degree. If no triangle exists, state "no triangle." If two triangles exist, solve each triangle.

1. $A = 70°, B = 55°, a = 12$
2. $B = 107°, C = 30°, c = 126$
3. $B = 66°, a = 17, c = 12$
4. $a = 117, b = 66, c = 142$
5. $A = 35°, B = 25°, c = 68$
6. $A = 39°, a = 20, b = 26$
7. $C = 50°, a = 3, c = 1$
8. $A = 162°, b = 11.2, c = 48.2$
9. $a = 26.1, b = 40.2, c = 36.5$
10. $A = 40°, a = 6, b = 4$
11. $B = 37°, a = 12.4, b = 8.7$
12. $A = 23°, a = 54.3, b = 22.1$

In Exercises 13–16, find the area of the triangle having the given measurements. Round to the nearest square unit.

13. $C = 42°$, $a = 4$ feet, $b = 6$ feet
14. $A = 22°$, $b = 4$ feet, $c = 5$ feet
15. $a = 2$ meters, $b = 4$ meters, $c = 5$ meters
16. $a = 2$ meters, $b = 2$ meters, $c = 2$ meters

DEFINITIONS AND CONCEPTS	EXAMPLES
e. If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.	Ex. 7, p. 1038
f. If A and B are not mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.	Ex. 8, p. 1039; Ex. 9, p. 1039
g. Two events are independent if the occurrence of either of them has no effect on the probability of the other.	
h. If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.	Ex. 10, p. 1041
i. The probability of a succession of independent events is the product of each of their probabilities.	Ex. 11, p. 1042

Review Exercises

11.1

In Exercises 1–6, write the first four terms of each sequence whose general term is given.

1. $a_n = 7n - 4$

2. $a_n = (-1)^n \dfrac{n+2}{n+1}$

3. $a_n = \dfrac{1}{(n-1)!}$

4. $a_n = \dfrac{(-1)^{n+1}}{2^n}$

5. $a_1 = 9$ and $a_n = \dfrac{2}{3a_{n-1}}$ for $n \geq 2$

6. $a_1 = 4$ and $a_n = 2a_{n-1} + 3$ for $n \geq 2$

7. Evaluate: $\dfrac{40!}{4!38!}$.

In Exercises 8–9, find each indicated sum.

8. $\displaystyle\sum_{i=1}^{5} (2i^2 - 3)$

9. $\displaystyle\sum_{i=0}^{4} (-1)^{i+1} i!$

In Exercises 10–11, express each sum using summation notation. Use i for the index of summation.

10. $\dfrac{1}{3} + \dfrac{2}{4} + \dfrac{3}{5} + \cdots + \dfrac{15}{17}$

11. $4^3 + 5^3 + 6^3 + \cdots + 13^3$

11.2

In Exercises 12–15, write the first six terms of each arithmetic sequence.

12. $a_1 = 7, d = 4$

13. $a_1 = -4, d = -5$

14. $a_1 = \frac{3}{2}, d = -\frac{1}{2}$

15. $a_{n+1} = a_n + 5, a_1 = -2$

In Exercises 16–18, find the indicated term of the arithmetic sequence with first term, a_1, and common difference, d.

16. Find a_6 when $a_1 = 5, d = 3$.

17. Find a_{12} when $a_1 = -8, d = -2$.

18. Find a_{14} when $a_1 = 14, d = -4$.

In Exercises 19–21, write a formula for the general term (the nth term) of each arithmetic sequence. Do not use a recursion formula. Then use the formula for a_n to find a_{20}, the 20th term of the sequence.

19. $-7, -3, 1, 5, \ldots$

20. $a_1 = 200, d = -20$

21. $a_n = a_{n-1} - 5, a_1 = 3$

22. Find the sum of the first 22 terms of the arithmetic sequence: 5, 12, 19, 26, ...

23. Find the sum of the first 15 terms of the arithmetic sequence: −6, −3, 0, 3, ...

24. Find $3 + 6 + 9 + \cdots + 300$, the sum of the first 100 positive multiples of 3.

In Exercises 25–27, use the formula for the sum of the first n terms of an arithmetic sequence to find the indicated sum.

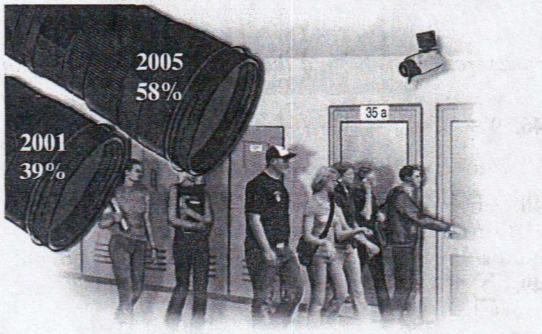
25. $\displaystyle\sum_{i=1}^{16} (3i + 2)$

26. $\displaystyle\sum_{i=1}^{25} (-2i + 6)$

27. $\displaystyle\sum_{i=1}^{30} (-5i)$

28. The graphic indicates that there are more eyes at school.

Percentage of United States Students Ages 12–18 Seeing Security Cameras at School



Source: Department of Education

In 2001, 39% of students ages 12–18 reported seeing one or more security cameras at their school. On average, this has increased by approximately 4.75% per year since then.

a. Write a formula for the nth term of the arithmetic sequence that describes the percentage of students ages 12–18 who reported seeing security cameras at school n years after 2000.

b. Use the model to predict the percentage of students ages 12–18 who will report seeing security cameras at school by the year 2013.

29. A company offers a starting salary of \$31,500 with raises of \$2300 per year. Find the total salary over a ten-year period.

30. A theater has 25 seats in the first row and 35 rows in all. Each successive row contains one additional seat. How many seats are in the theater?

11.3

In Exercises 31–34, write the first five terms of each geometric sequence.

31. $a_1 = 3, r = 2$

32. $a_1 = \frac{1}{2}, r = \frac{1}{2}$

33. $a_1 = 16, r = -\frac{1}{2}$

34. $a_n = -5a_{n-1}, a_1 = -1$

In Exercises 35–37, use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of each sequence.

35. Find a_7 when $a_1 = 2, r = 3$.

36. Find a_6 when $a_1 = 16, r = \frac{1}{2}$.

37. Find a_5 when $a_1 = -3, r = 2$.

In Exercises 38–40, write a formula for the general term (the nth term) of each geometric sequence. Then use the formula for a_n to find a_8, the eighth term of the sequence.

38. $1, 2, 4, 8, \ldots$

39. $100, 10, 1, \frac{1}{10}, \ldots$

40. $12, -4, \frac{4}{3}, -\frac{4}{9}, \ldots$

41. Find the sum of the first 15 terms of the geometric sequence: $5, -15, 45, -135, \ldots$.

42. Find the sum of the first 7 terms of the geometric sequence: $8, 4, 2, 1, \ldots$.

In Exercises 43–45, use the formula for the sum of the first n terms of a geometric sequence to find the indicated sum.

43. $\sum_{i=1}^{6} 5^i$

44. $\sum_{i=1}^{7} 3(-2)^i$

45. $\sum_{i=1}^{5} 2\left(\frac{1}{4}\right)^{i-1}$

In Exercises 46–49, find the sum of each infinite geometric series.

46. $9 + 3 + 1 + \frac{1}{3} + \cdots$

47. $2 - 1 + \frac{1}{2} - \frac{1}{4} + \cdots$

48. $-6 + 4 - \frac{8}{3} + \frac{16}{9} - \cdots$

49. $\sum_{i=1}^{\infty} 5(0.8)^i$

In Exercises 50–51, express each repeating decimal as a fraction in lowest terms.

50. $0.\overline{6}$

51. $0.\overline{47}$

52. Projections for the U.S. population, ages 85 and older, are shown in the following table.

Year	2000	2010	2020	2030	2040	2050
Projected Population in millions	4.2	5.9	8.3	11.6	16.2	22.7

Actual 2000 population

Source: U.S. Census Bureau

a. Show that the U.S. population, ages 85 and older, is projected to increase geometrically.

b. Write the general term of the geometric sequence describing the U.S. population ages 85 and older, in millions, n decades after 2000.

c. Use the model in part (b) to project the U.S. population, ages 85 and older, in 2080.

53. A job pays \$32,000 for the first year with an annual increase of 6% per year beginning in the second year. What is the salary in the sixth year? What is the total salary paid over this six-year period? Round answers to the nearest dollar.

In Exercises 54–55, use the formula for the value of an annuity and round to the nearest dollar.

54. You spend \$10 per week on lottery tickets, averaging \$520 per year. Instead of buying tickets, if you deposited the \$520 at the end of each year in an annuity paying 6% compounded annually,

a. How much would you have after 20 years?

b. Find the interest.

55. To save for retirement, you decide to deposit \$100 at the end of each month in an IRA that pays 5.5% compounded monthly.

a. How much will you have from the IRA after 30 years?

b. Find the interest.

56. A factory in an isolated town has an annual payroll of \$4 million. It is estimated that 70% of this money is spent within the town, that people in the town receiving this money will again spend 70% of what they receive in the town, and so on. What is the total of all this spending in the town each year?

11.4

In Exercises 57–61, use mathematical induction to prove that each statement is true for every positive integer n.

57. $5 + 10 + 15 + \cdots + 5n = \frac{5n(n+1)}{2}$

58. $1 + 4 + 4^2 + \cdots + 4^{n-1} = \frac{4^n - 1}{3}$

59. $2 + 6 + 10 + \cdots + (4n - 2) = 2n^2$

60. $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}$

61. 2 is a factor of $n^2 + 5n$.

11.5

In Exercises 62–63, evaluate the given binomial coefficient.

62. $\binom{11}{8}$

63. $\binom{90}{2}$

In Exercises 64–67, use the Binomial Theorem to expand each binomial and express the result in simplified form.

64. $(2x + 1)^3$

65. $(x^2 - 1)^4$

66. $(x + 2y)^5$

67. $(x - 2)^6$

In Exercises 68–69, write the first three terms in each binomial expansion, expressing the result in simplified form.

68. $(x^2 + 3)^8$ **69.** $(x - 3)^9$

In Exercises 70–71, find the term indicated in each expansion.

70. $(x + 2)^5$; fourth term **71.** $(2x - 3)^6$; fifth term

11.6

In Exercises 72–75, evaluate each expression.

72. ${}_8P_3$ **73.** ${}_9P_5$

74. ${}_8C_3$ **75.** ${}_{13}C_{11}$

In Exercises 76–82, solve by the method of your choice.

76. A popular brand of pen comes in red, green, blue, or black ink. The writing tip can be chosen from extra bold, bold, regular, fine, or micro. How many different choices of pens do you have with this brand?

77. A stock can go up, go down, or stay unchanged. How many possibilities are there if you own five stocks?

78. A club with 15 members is to choose four officers—president, vice-president, secretary, and treasurer. In how many ways can these offices be filled?

79. How many different ways can a director select 4 actors from a group of 20 actors to attend a workshop on performing in rock musicals?

80. From the 20 CDs that you've bought during the past year, you plan to take 3 with you on vacation. How many different sets of three CDs can you take?

81. How many different ways can a director select from 20 male actors and cast the roles of Mark, Roger, Angel, and Collins in the musical *Rent*?

82. In how many ways can five airplanes line up for departure on a runway?

11.7

Suppose that a survey of 350 college students is taken. Each student is asked the type of college attended (public or private) and the family's income level (low, middle, high). Use the data in the table to solve Exercises 83–88. Express probabilities as simplified fractions.

	Public	Private	Total
Low	120	20	140
Middle	110	50	160
High	22	28	50
Total	252	98	350

Find the probability that a randomly selected student in the survey

83. attends a public college.

84. is not from a high-income family.

85. is from a middle-income or a high-income family.

86. attends a private college or is from a high-income family.

87. Among people who attend a public college, find the probability that a randomly selected student is from a low-income family.

88. Among people from a middle-income family, find the probability that a randomly selected student attends a private college.

In Exercises 89–90, a die is rolled. Find the probability of

89. getting a number less than 5.

90. getting a number less than 3 or greater than 4.

In Exercises 91–92, you are dealt one card from a 52-card deck. Find the probability of

91. getting an ace or a king.

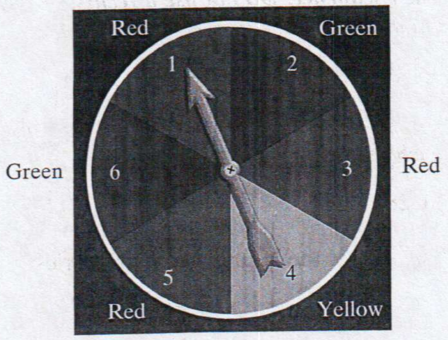
92. getting a queen or a red card.

In Exercises 93–95, it is equally probable that the pointer on the spinner shown will land on any one of the six regions, numbered 1 through 6, and colored as shown. If the pointer lands on a borderline, spin again. Find the probability of

93. not stopping on yellow.

94. stopping on red or a number greater than 3.

95. stopping on green on the first spin and stopping on a number less than 4 on the second spin.



96. A lottery game is set up so that each player chooses five different numbers from 1 to 20. If the five numbers match the five numbers drawn in the lottery, the player wins (or shares) the top cash prize. What is the probability of winning the prize

a. with one lottery ticket?

b. with 100 different lottery tickets?

97. What is the probability of a family having five boys born in a row?

98. The probability of a flood in any given year in a region prone to floods is 0.2.

a. What is the probability of a flood two years in a row?

b. What is the probability of a flood for three consecutive years?

c. What is the probability of no flooding for four consecutive years?