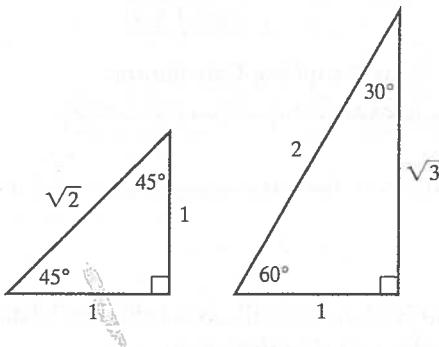


In Exercises 9–16, use the given triangles to evaluate each expression. If necessary, express the value without a square root in the denominator by rationalizing the denominator.



9. $\cos 30^\circ$
10. $\tan 30^\circ$
11. $\sec 45^\circ$
12. $\csc 45^\circ$
13. $\tan \frac{\pi}{3}$
14. $\cot \frac{\pi}{3}$
15. $\sin \frac{\pi}{4} - \cos \frac{\pi}{4}$
16. $\tan \frac{\pi}{4} + \csc \frac{\pi}{6}$

In Exercises 17–20, θ is an acute angle and $\sin \theta$ and $\cos \theta$ are given. Use identities to find $\tan \theta$, $\csc \theta$, $\sec \theta$, and $\cot \theta$. Where necessary, rationalize denominators.

17. $\sin \theta = \frac{8}{17}$, $\cos \theta = \frac{15}{17}$
18. $\sin \theta = \frac{3}{5}$, $\cos \theta = \frac{4}{5}$
19. $\sin \theta = \frac{1}{3}$, $\cos \theta = \frac{2\sqrt{2}}{3}$
20. $\sin \theta = \frac{6}{7}$, $\cos \theta = \frac{\sqrt{13}}{7}$

In Exercises 21–24, θ is an acute angle and $\sin \theta$ is given. Use the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ to find $\cos \theta$.

21. $\sin \theta = \frac{6}{7}$
22. $\sin \theta = \frac{7}{8}$
23. $\sin \theta = \frac{\sqrt{39}}{8}$
24. $\sin \theta = \frac{\sqrt{21}}{5}$

In Exercises 25–30, use an identity to find the value of each expression. Do not use a calculator.

25. $\sin 37^\circ \csc 37^\circ$
26. $\cos 53^\circ \sec 53^\circ$
27. $\sin^2 \frac{\pi}{9} + \cos^2 \frac{\pi}{9}$
28. $\sin^2 \frac{\pi}{10} + \cos^2 \frac{\pi}{10}$
29. $\sec^2 23^\circ - \tan^2 23^\circ$
30. $\csc^2 63^\circ - \cot^2 63^\circ$

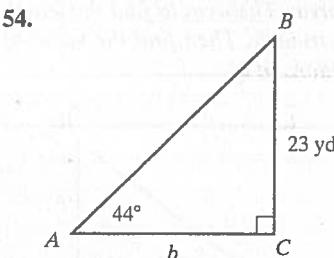
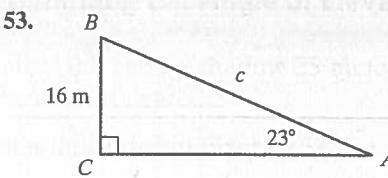
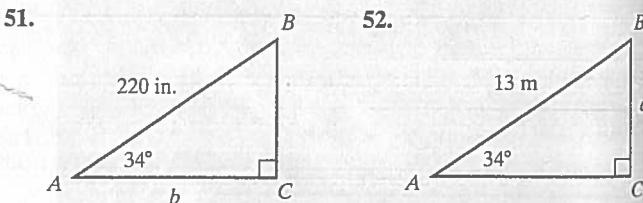
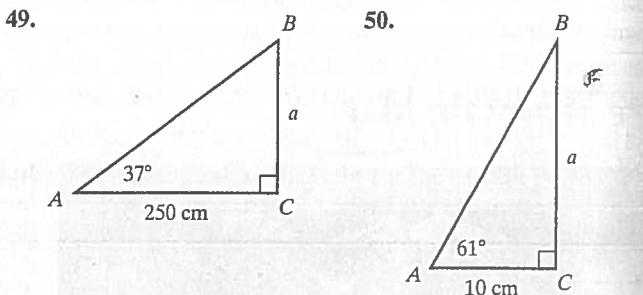
In Exercises 31–38, find a cofunction with the same value as the given expression.

31. $\sin 7^\circ$
32. $\sin 19^\circ$
33. $\csc 25^\circ$
34. $\csc 35^\circ$
35. $\tan \frac{\pi}{9}$
36. $\tan \frac{\pi}{7}$
37. $\cos \frac{2\pi}{5}$
38. $\cos \frac{3\pi}{8}$

In Exercises 39–48, use a calculator to find the value of the trigonometric function to four decimal places.

39. $\sin 38^\circ$
40. $\cos 21^\circ$
41. $\tan 32.7^\circ$
42. $\tan 52.6^\circ$
43. $\csc 17^\circ$
44. $\sec 55^\circ$
45. $\cos \frac{\pi}{10}$
46. $\sin \frac{3\pi}{10}$
47. $\cot \frac{\pi}{12}$
48. $\cot \frac{\pi}{18}$

In Exercises 49–54, find the measure of the side of the right triangle whose length is designated by a lowercase letter. Round answers to the nearest whole number.



In Exercises 55–58, use a calculator to find the value of the acute angle θ to the nearest degree.

55. $\sin \theta = 0.2974$
56. $\cos \theta = 0.8771$
57. $\tan \theta = 4.6252$
58. $\tan \theta = 26.0307$

In Exercises 59–62, use a calculator to find the value of the acute angle θ in radians, rounded to three decimal places.

59. $\cos \theta = 0.4112$
60. $\sin \theta = 0.9499$
61. $\tan \theta = 0.4169$
62. $\tan \theta = 0.5117$

Exercise Set 5.3

Practice Exercises

In Exercises 1–8, a point on the terminal side of angle θ is given. Find the exact value of each of the six trigonometric functions of θ .

- | | | |
|---------------|---------------|--------------|
| 1. $(-4, 3)$ | 2. $(-12, 5)$ | 3. $(2, 3)$ |
| 4. $(3, 7)$ | 5. $(3, -3)$ | 6. $(5, -5)$ |
| 7. $(-2, -5)$ | 8. $(-1, -3)$ | |

In Exercises 9–16, evaluate the trigonometric function at the quadrantal angle, or state that the expression is undefined.

- | | | |
|--------------------------|---------------------------|---------------------------|
| 9. $\cos \pi$ | 10. $\tan \pi$ | 11. $\sec \pi$ |
| 12. $\csc \pi$ | 13. $\tan \frac{3\pi}{2}$ | 14. $\cos \frac{3\pi}{2}$ |
| 15. $\cot \frac{\pi}{2}$ | 16. $\tan \frac{\pi}{2}$ | |

In Exercises 17–22, let θ be an angle in standard position. Name the quadrant in which θ lies.

- | | |
|--|--|
| 17. $\sin \theta > 0, \cos \theta > 0$ | 18. $\sin \theta < 0, \cos \theta > 0$ |
| 19. $\sin \theta < 0, \cos \theta < 0$ | 20. $\tan \theta < 0, \sin \theta < 0$ |
| 21. $\tan \theta < 0, \cos \theta < 0$ | 22. $\cot \theta > 0, \sec \theta < 0$ |

In Exercises 23–34, find the exact value of each of the remaining trigonometric functions of θ .

- | | |
|--|---|
| 23. $\cos \theta = -\frac{3}{5}, \theta$ in quadrant III | |
| 24. $\sin \theta = -\frac{12}{13}, \theta$ in quadrant III | |
| 25. $\sin \theta = \frac{5}{13}, \theta$ in quadrant II | |
| 26. $\cos \theta = \frac{4}{5}, \theta$ in quadrant IV | |
| 27. $\cos \theta = \frac{8}{17}, 270^\circ < \theta < 360^\circ$ | |
| 28. $\cos \theta = \frac{1}{3}, 270^\circ < \theta < 360^\circ$ | |
| 29. $\tan \theta = -\frac{2}{3}, \sin \theta > 0$ | 30. $\tan \theta = -\frac{1}{3}, \sin \theta > 0$ |
| 31. $\tan \theta = \frac{4}{3}, \cos \theta < 0$ | 32. $\tan \theta = \frac{5}{12}, \cos \theta < 0$ |
| 33. $\sec \theta = -3, \tan \theta > 0$ | 34. $\csc \theta = -4, \tan \theta > 0$ |

In Exercises 35–60, find the reference angle for each angle.

- | | | |
|--------------------|------------------------|------------------------|
| 35. 160° | 36. 170° | 37. 205° |
| 38. 210° | 39. 355° | 40. 351° |
| $\frac{7\pi}{4}$ | $\frac{5\pi}{4}$ | $\frac{5\pi}{6}$ |
| $\frac{5\pi}{7}$ | 45. -150° | 46. -250° |
| 47. -335° | 48. -359° | 49. 4.7 |
| 50. 5.5 | 51. 565° | 52. 553° |
| $\frac{17\pi}{6}$ | $\frac{11\pi}{4}$ | $\frac{23\pi}{4}$ |
| $\frac{17\pi}{3}$ | 57. $-\frac{11\pi}{4}$ | 58. $-\frac{17\pi}{6}$ |
| $-\frac{25\pi}{6}$ | 60. $-\frac{13\pi}{3}$ | |

In Exercises 61–86, use reference angles to find the exact value of each expression. Do not use a calculator.

- | | | |
|----------------------|----------------------|----------------------|
| 61. $\cos 225^\circ$ | 62. $\sin 300^\circ$ | 63. $\tan 210^\circ$ |
| 64. $\sec 240^\circ$ | 65. $\tan 420^\circ$ | 66. $\tan 405^\circ$ |

- | | | |
|---|---|---|
| 67. $\sin \frac{2\pi}{3}$ | 68. $\cos \frac{3\pi}{4}$ | 69. $\csc \frac{7\pi}{6}$ |
| 70. $\cot \frac{7\pi}{4}$ | 71. $\tan \frac{9\pi}{4}$ | 72. $\tan \frac{9\pi}{2}$ |
| 73. $\sin(-240^\circ)$ | 74. $\sin(-225^\circ)$ | 75. $\tan\left(-\frac{\pi}{4}\right)$ |
| 76. $\tan\left(-\frac{\pi}{6}\right)$ | 77. $\sec 495^\circ$ | 78. $\sec 510^\circ$ |
| 79. $\cot \frac{19\pi}{6}$ | 80. $\cot \frac{13\pi}{3}$ | 81. $\cos \frac{23\pi}{4}$ |
| 82. $\cos \frac{35\pi}{6}$ | 83. $\tan\left(-\frac{17\pi}{6}\right)$ | 84. $\tan\left(-\frac{11\pi}{4}\right)$ |
| 85. $\sin\left(-\frac{17\pi}{3}\right)$ | 86. $\sin\left(-\frac{35\pi}{6}\right)$ | |

Practice Plus

In Exercises 87–92, find the exact value of each expression. Write the answer as a single fraction. Do not use a calculator.

- | |
|--|
| 87. $\sin \frac{\pi}{3} \cos \pi - \cos \frac{\pi}{3} \sin \frac{3\pi}{2}$ |
| 88. $\sin \frac{\pi}{4} \cos 0 - \sin \frac{\pi}{6} \cos \frac{\pi}{3}$ |
| 89. $\sin \frac{11\pi}{4} \cos \frac{5\pi}{6} + \cos \frac{11\pi}{4} \sin \frac{5\pi}{6}$ |
| 90. $\sin \frac{17\pi}{3} \cos \frac{5\pi}{4} + \cos \frac{17\pi}{3} \sin \frac{5\pi}{4}$ |
| 91. $\sin \frac{3\pi}{2} \tan\left(-\frac{15\pi}{4}\right) - \cos\left(-\frac{5\pi}{3}\right)$ |
| 92. $\sin \frac{3\pi}{2} \tan\left(-\frac{8\pi}{3}\right) + \cos\left(-\frac{5\pi}{6}\right)$ |

In Exercises 93–98, let

$$f(x) = \sin x, g(x) = \cos x, \text{ and } h(x) = 2x.$$

Find the exact value of each expression. Do not use a calculator.

- | | |
|---|---|
| 93. $f\left(\frac{4\pi}{3} + \frac{\pi}{6}\right) + f\left(\frac{4\pi}{3}\right) + f\left(\frac{\pi}{6}\right)$ | |
| 94. $g\left(\frac{5\pi}{6} + \frac{\pi}{6}\right) + g\left(\frac{5\pi}{6}\right) + g\left(\frac{\pi}{6}\right)$ | |
| 95. $(h \circ g)\left(\frac{17\pi}{3}\right)$ | 96. $(h \circ f)\left(\frac{11\pi}{4}\right)$ |

- | |
|---|
| 97. the average rate of change of f from $x_1 = \frac{5\pi}{4}$ to $x_2 = \frac{3\pi}{2}$ |
| 98. the average rate of change of g from $x_1 = \frac{3\pi}{4}$ to $x_2 = \pi$ |

In Exercises 99–104, find two values of θ , $0 \leq \theta < 2\pi$, that satisfy each equation.

- | | |
|--|--|
| 99. $\sin \theta = \frac{\sqrt{2}}{2}$ | 100. $\cos \theta = \frac{1}{2}$ |
| 101. $\sin \theta = -\frac{\sqrt{2}}{2}$ | 102. $\cos \theta = -\frac{1}{2}$ |
| 103. $\tan \theta = -\sqrt{3}$ | 104. $\tan \theta = -\frac{\sqrt{3}}{3}$ |

Study Tip

Much of the essential information in this chapter can be found in three places:

- Study Tip on page 524, showing special angles and how to obtain exact values of trigonometric functions at these angles
- Table 5.6 on page 566, showing the graphs of the six trigonometric functions, with their domains, ranges, and periods
- Table 5.10 on page 578, showing graphs of the three basic inverse trigonometric functions, with their domains and ranges.

Make copies of these pages and mount them on cardstock. Use this reference sheet as you work the review exercises until you have all the information on the reference sheet memorized for the chapter test.

Review Exercises**5.1**

1. Find the radian measure of the central angle of a circle of radius 6 centimeters that intercepts an arc of length 27 centimeters.

In Exercises 2–4, convert each angle in degrees to radians. Express your answer as a multiple of π .

2. 15° 3. 120° 4. 315°

In Exercises 5–7, convert each angle in radians to degrees.

5. $\frac{5\pi}{3}$ 6. $\frac{7\pi}{5}$ 7. $-\frac{5\pi}{6}$

In Exercises 8–12, draw each angle in standard position.

8. $\frac{5\pi}{6}$ 9. $-\frac{2\pi}{3}$ 10. $\frac{8\pi}{3}$
11. 190° 12. -135°

In Exercises 13–17, find a positive angle less than 360° or 2π that is coterminal with the given angle.

13. 400° 14. -445° 15. $\frac{13\pi}{4}$
16. $\frac{31\pi}{6}$ 17. $-\frac{8\pi}{3}$

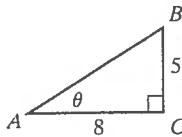
18. Find the length of the arc on a circle of radius 10 feet intercepted by a 135° central angle. Express arc length in terms of π . Then round your answer to two decimal places.

19. The angular speed of a propeller on a wind generator is 10.3 revolutions per minute. Express this angular speed in radians per minute.

20. The propeller of an airplane has a radius of 3 feet. The propeller is rotating at 2250 revolutions per minute. Find the linear speed, in feet per minute, of the tip of the propeller.

5.2

21. Use the triangle to find each of the six trigonometric functions of θ .



In Exercises 22–25, find the exact value of each expression. Do not use a calculator.

22. $\sin \frac{\pi}{6} + \tan^2 \frac{\pi}{3}$ 23. $\cos^2 \frac{\pi}{4} - \tan^2 \frac{\pi}{4}$

24. $\sec^2 \frac{\pi}{5} - \tan^2 \frac{\pi}{5}$ 25. $\cos \frac{2\pi}{9} \sec \frac{2\pi}{9}$

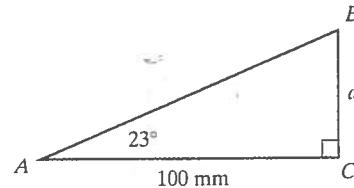
26. If θ is an acute angle and $\sin \theta = \frac{2\sqrt{7}}{7}$, use the identity $\sin^2 \theta + \cos^2 \theta = 1$ to find $\cos \theta$.

In Exercises 27–28, find a cofunction with the same value as the given expression.

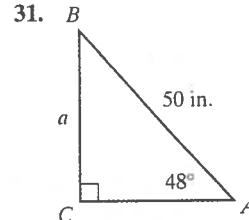
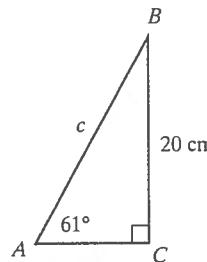
27. $\sin 70^\circ$ 28. $\cos \frac{\pi}{2}$

In Exercises 29–31, find the measure of the side of the right triangle whose length is designated by a lowercase letter. Round answers to the nearest whole number.

29.



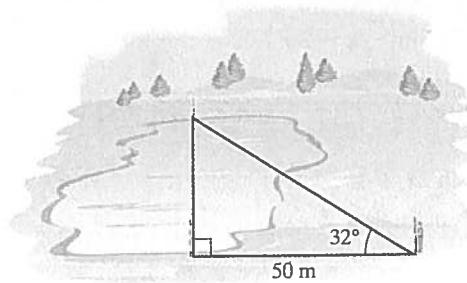
30.



32. If $\sin \theta = \frac{1}{4}$ and θ is acute, find $\tan \left(\frac{\pi}{2} - \theta \right)$.

33. A hiker climbs for a half mile up a slope whose inclination is 17° . How many feet of altitude, to the nearest foot, does the hiker gain?

34. To find the distance across a lake, a surveyor took the measurements in the figure shown. What is the distance across the lake? Round to the nearest meter.



35. When a six-foot pole casts a four-foot shadow, what is the angle of elevation of the sun? Round to the nearest whole degree.

5.3 and 5.4

In Exercises 36–37, a point on the terminal side of angle θ is given. Find the exact value of each of the six trigonometric functions of θ , or state that the function is undefined.

36. $(-1, -5)$

37. $(0, -1)$

In Exercises 38–39, let θ be an angle in standard position. Name the quadrant in which θ lies.

38. $\tan \theta > 0$ and $\sec \theta > 0$

39. $\tan \theta > 0$ and $\cos \theta < 0$

In Exercises 40–42, find the exact value of each of the remaining trigonometric functions of θ .

40. $\cos \theta = \frac{2}{5}$, $\sin \theta < 0$

41. $\tan \theta = -\frac{1}{3}$, $\sin \theta > 0$

42. $\cot \theta = 3$, $\cos \theta < 0$

In Exercises 43–47, find the reference angle for each angle.

43. 265°

44. $\frac{5\pi}{8}$

45. -410°

46. $\frac{17\pi}{6}$

47. $-\frac{11\pi}{3}$

In Exercises 48–58, find the exact value of each expression. Do not use a calculator.

48. $\sin 240^\circ$

49. $\tan 120^\circ$

50. $\sec \frac{7\pi}{4}$

51. $\cos \frac{11\pi}{6}$

52. $\cot(-210^\circ)$

53. $\csc\left(-\frac{2\pi}{3}\right)$

54. $\sin\left(-\frac{\pi}{3}\right)$

55. $\sin 495^\circ$

56. $\tan \frac{13\pi}{4}$

57. $\sin \frac{22\pi}{3}$

58. $\cos\left(-\frac{35\pi}{6}\right)$

5.5

In Exercises 59–64, determine the amplitude and period of each function. Then graph one period of the function.

59. $y = 3 \sin 4x$

60. $y = -2 \cos 2x$

61. $y = 2 \cos \frac{1}{2}x$

62. $y = \frac{1}{2} \sin \frac{\pi}{3}x$

63. $y = -\sin \pi x$

64. $y = 3 \cos \frac{x}{3}$

In Exercises 65–69, determine the amplitude, period, and phase shift of each function. Then graph one period of the function.

65. $y = 2 \sin(x - \pi)$

66. $y = -3 \cos(x + \pi)$

67. $y = \frac{3}{2} \cos\left(2x + \frac{\pi}{4}\right)$

68. $y = \frac{5}{2} \sin\left(2x + \frac{\pi}{2}\right)$

69. $y = -3 \sin\left(\frac{\pi}{3}x - 3\pi\right)$

In Exercises 70–71, use a vertical shift to graph one period of the function.

70. $y = \sin 2x + 1$

71. $y = 2 \cos \frac{1}{3}x - 2$

72. The function

$$y = 98.6 + 0.3 \sin\left(\frac{\pi}{12}x - \frac{11\pi}{12}\right)$$

models variation in body temperature, y , in $^{\circ}\text{F}$, x hours after midnight.

a. What is body temperature at midnight?

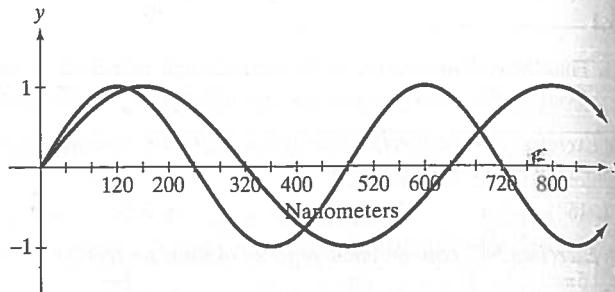
b. What is the period of the body temperature cycle?

c. When is body temperature highest? What is the body temperature at this time?

d. When is body temperature lowest? What is the body temperature at this time?

e. Graph one period of the body temperature function.

73. Light waves can be modeled by sine functions. The graphs show waves of red and blue light. Write an equation in the form $y = A \sin Bx$ that models each of these light waves.

**5.6**

In Exercises 74–80, graph two full periods of the given tangent or cotangent function.

74. $y = 4 \tan 2x$

75. $y = -2 \tan \frac{\pi}{4}x$

76. $y = \tan(x + \pi)$

77. $y = -\tan\left(x - \frac{\pi}{4}\right)$

78. $y = 2 \cot 3x$

79. $y = -\frac{1}{2} \cot \frac{\pi}{2}x$

80. $y = 2 \cot\left(x + \frac{\pi}{2}\right)$

In Exercises 81–84, graph two full periods of the given cosecant or secant function.

81. $y = 3 \sec 2\pi x$

82. $y = -2 \csc \pi x$

83. $y = 3 \sec(x + \pi)$

84. $y = \frac{5}{2} \csc(x - \pi)$

5.7

In Exercises 85–103, find the exact value of each expression. Do not use a calculator.

85. $\sin^{-1} 1$

86. $\cos^{-1} 1$

87. $\tan^{-1} 1$

88. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

89. $\cos^{-1}\left(-\frac{1}{2}\right)$

90. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

91. $\cos\left(\sin^{-1} \frac{\sqrt{2}}{2}\right)$

92. $\sin(\cos^{-1} 0)$

93. $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$

94. $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

95. $\csc\left(\tan^{-1} \frac{\sqrt{3}}{3}\right)$

96. $\cos\left(\tan^{-1} \frac{3}{4}\right)$