#### **DEFINITIONS AND CONCEPTS**

### **EXAMPLES**

d. Linear systems can be represented by matrix equations of the form AX = B in which A is the coefficient matrix and B is the constant matrix. If AX = B has a unique solution, then  $X = A^{-1}B$ .

Ex. 5, p. 867

### 9.5 Determinants and Cramer's Rule

a. Value of a second-order determinant:

Ex. 1, p. 873

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

b. Cramer's rule for solving systems of linear equations in two variables uses three second-order determinants and is stated in the box on page 874.

Ex. 2, p. 875

c. To evaluate an *n*th-order determinant, where n > 2.

Ex. 3, p. 876;

1. Select a row or column about which to expand.

Ex. 4, p. 878;

2. Multiply each element  $a_{ij}$  in the row or column by  $(-1)^{i+j}$  times the determinant obtained by deleting the ith row and the jth column in the given array of numbers.

Ex. 6, p. 881

- 3. The value of the determinant is the sum of the products found in step 2.
- d. Cramer's rule for solving systems of linear equations in three variables uses four third-order determinants and is stated in the box on page 878.

Ex. 5, p. 879

e. Cramer's rule with inconsistent and dependent systems is summarized by the two situations in the box on page 880.

# **Review Exercises**

## 9.1

*In Exercises 1–2, perform each matrix row operation and write the* new matrix.

1. 
$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 5 & 4 & 1 \end{bmatrix} -5R_2 + R_3$$

**2.** 
$$\begin{bmatrix} 2 & -2 & 1 & -1 \\ 1 & 2 & -1 & 2 \\ 6 & 4 & 3 & 5 \end{bmatrix} \quad \frac{1}{2}R_1$$

In Exercises 3-5, solve each system of equations using matrices. Use Gaussian elimination with back-substitution or Gauss-Jordan elimination

3. 
$$\begin{cases} x + 2y + 3z = -5 \\ 2x + y + z = 1 \end{cases}$$

3. 
$$\begin{cases} x + 2y + 3z = -5 \\ 2x + y + z = 1 \\ x + y - z = 8 \end{cases}$$
4. 
$$\begin{cases} x - 2y + z = 0 \\ y - 3z = -1 \\ 2y + 5z = -2 \end{cases}$$

5. 
$$\begin{cases} 3x_1 + 5x_2 - 8x_3 + 5x_4 = -8 \\ x_1 + 2x_2 - 3x_3 + x_4 = -7 \\ 2x_1 + 3x_2 - 7x_3 + 3x_4 = -11 \end{cases}$$

$$2x_1 + 3x_2 - 7x_3 + 3x_4 = -11$$

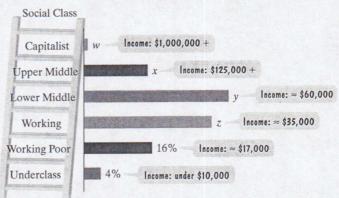
$$4x_1 + 8x_2 - 10x_3 + 7x_4 = -10$$

6. The table shows the pollutants in the air in a city on a typical summer day.

x (Hours after 6 A.M.)	y (Amount of Pollutants in the Air, in parts per million)
2	98
4	138
10	162

- **a.** Use the function  $y = ax^2 + bx + c$  to model the data. Use either Gaussian elimination with back-substitution or Gauss-Jordan elimination to find the values for a, b, and c.
- b. Use the function to find the time of day at which the city's air pollution level is at a maximum. What is the maximum level?
- 7. Sociologists Joseph Kahl and Dennis Gilbert developed a six-tier model to portray the class structure of the United States. The bar graph represents the percentage of Americans who are members of each of the six social classes.

#### The United States Social Class Ladder



Percentage of the Population

Source: James Henslin, Sociology, Eighth Edition, Allyn and Bacon, 2007

Combined, members of the capitalist class, the upper-middle class, the lower-middle class, and the working class make up 80% of the U.S. population. The percentage of the population belonging to the lower-middle class exceeds the percentage belonging to capitalist and upper-middle classes by 18%. The difference between the percentage belonging to the lowermiddle class and the working class is 4%. If the percentage belonging to the upper-middle class is tripled, it exceeds the percentage belonging to the capitalist and lower-middle classes by 10%. Determine the percentage of the U.S. population who are members of the capitalist class, the upper-middle class, the lower-middle class, and the working class.

# 9.2

In Exercises 8-11, use Gaussian elimination to find the complete solution to each system, or show that none exists.

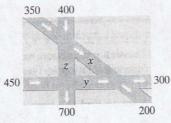
8. 
$$\begin{cases} 2x - 3y + z = 1\\ x - 2y + 3z = 2\\ 3x - 4y - z = 1 \end{cases}$$

8. 
$$\begin{cases} 2x - 3y + z = 1 \\ x - 2y + 3z = 2 \\ 3x - 4y - z = 1 \end{cases}$$
9. 
$$\begin{cases} x - 3y + z = 1 \\ -2x + y + 3z = -7 \\ x - 4y + 2z = 0 \end{cases}$$
 levels shown.

10. 
$$\begin{cases} x_1 + 4x_2 + 3x_3 - 6x_4 = 5 \\ x_1 + 3x_2 + x_3 - 4x_4 = 3 \\ 2x_1 + 8x_2 + 7x_3 - 5x_4 = 11 \\ 2x_1 + 5x_2 - 6x_4 = 4 \end{cases}$$

11. 
$$\begin{cases} 2x + 3y - 5z = 15 \\ x + 2y - z = 4 \end{cases}$$

12. The figure shows the intersections of three one-way streets. The numbers given represent traffic flow, in cars per hour, at a peak period (from 4 P.M. to 6 P.M.).



- a. Use the idea that the number of cars entering each intersection per hour must equal the number of cars leaving per hour to set up a system of linear equations involving x, y, and z.
- b. Use Gaussian elimination to solve the system.
- c. If construction limits the value of z to 400, how many cars per hour must pass between the other intersections to keep traffic flowing?

# 9.3

13. Find values for x, y, and z so that the following matrices are

$$\begin{bmatrix} 2x & y+7 \\ z & 4 \end{bmatrix} = \begin{bmatrix} -10 & 13 \\ 6 & 4 \end{bmatrix}.$$

In Exercises 14-27, perform the indicated matrix operations given that A, B, C, and D are defined as follows. If an operation is not defined, state the reason.

$$A = \begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -2 \\ 3 & 2 \\ 1 & -5 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix} \quad D = \begin{bmatrix} -2 & 3 & 1 \\ 3 & -2 & 4 \end{bmatrix}$$

14. 
$$A + D$$

**16.** 
$$D - A$$

17. 
$$B + C$$

18. 
$$3A + 2D$$

$$19 - 2A + 4D$$

**20.** 
$$-5(A+D)$$
 **21.**  $AB$ 

25. 
$$AB - BA$$

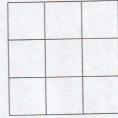
**26.** 
$$(A - D)C$$

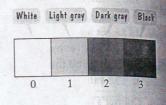
28. Solve for X in the matrix equation

$$3X + A = B,$$

where 
$$A = \begin{bmatrix} 4 & 6 \\ -5 & 0 \end{bmatrix}$$
 and  $B = \begin{bmatrix} -2 & -12 \\ 4 & 1 \end{bmatrix}$ .

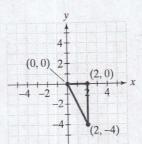
In Exercises 29-30, use nine pixels in a 3 × 3 grid and the color





- 29. Write a  $3 \times 3$  matrix that represents a digital photograph of the letter T in dark gray on a light gray background.
- 30. Find a matrix B so that A + B increases the contrast of the letter T by changing the dark gray to black and the light gray to white.

The figure shows a right triangle in a rectangular coordinate



The figure can be represented by the matrix

$$B = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -4 \end{bmatrix}$$

Use the triangle and the matrix that represents it to solve Exercises 31-36.

- 31. Use matrix operations to move the triangle 2 units to the left and 1 unit up. Then graph the triangle and its transformation in a rectangular coordinate system.
- 32. Use matrix operations to reduce the triangle to half its perimeter and move the reduced image 2 units down. Then graph the triangle and its transformation in a rectangular coordinate system.

In Exercises 33-36, find AB and graph the resulting image. What effect does the multiplication have on the triangle represented by

$$\mathbf{33.} \ A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

**34.** 
$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

**35.** 
$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
 **36.**  $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ 

**36.** 
$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

37. 
$$A = \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix}$$
,  $B = \begin{bmatrix} 4 & -7 \\ -1 & 3 \end{bmatrix}$ 

38. 
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 7 \\ 0 & 1 & 2 \end{bmatrix}$$

In Exercises 39–42, find  $A^{-1}$ . Check that  $AA^{-1} = I$  and  $A^{-1}A = I$ .

$$\mathbf{39.} \ \ A = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

**40.** 
$$A = \begin{bmatrix} 0 & 1 \\ 5 & 3 \end{bmatrix}$$

**41.** 
$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & -3 \end{bmatrix}$$

**41.** 
$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & -3 \end{bmatrix}$$
 **42.**  $A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 13 & -7 \\ 5 & 16 & -8 \end{bmatrix}$ 

In Exercises 43-44

- a. Write each linear system as a matrix equation in the form AX = B.
- b. Solve the system using the inverse that is given for the coefficient matrix.

43. 
$$\begin{cases} x + y + 2z = 7 \\ y + 3z = -2 \\ 3x - 2z = 0 \end{cases} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 3 & 0 & -2 \end{bmatrix}$$
 is 
$$\begin{bmatrix} -2 & 2 & 1 \\ 9 & -8 & -3 \\ -3 & 3 & 1 \end{bmatrix}$$
.

44. 
$$\begin{cases} x - y + 2z = 12 \\ y - z = -5 \\ x + 2z = 10 \end{cases} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix} \text{ is } \begin{bmatrix} 2 & 2 & -1 \\ -1 & 0 & 1 \\ -1 & -1 & 1 \end{bmatrix}.$$

**45.** Use the coding matrix  $A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$  and its inverse  $A^{-1} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$  to encode and then decode the word

## 9.5

In Exercises 46-51, evaluate each determinant.

**46.** 
$$\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix}$$

**47.** 
$$\begin{vmatrix} -2 & -3 \\ -4 & -8 \end{vmatrix}$$

$$\begin{array}{c|ccccc} 49. & 4 & 7 & 0 \\ -5 & 6 & 0 \\ 3 & 2 & -4 \end{array}$$

50. 
$$\begin{vmatrix} 1 & 1 & 0 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & -2 & 4 & 0 \\ 0 & 3 & 0 & 1 \end{vmatrix}$$

51. 
$$\begin{vmatrix} 2 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

In Exercises 52-55, use Cramer's rule to solve each system.

$$52. \begin{cases} x - 2y = 8 \\ 3x + 2y = -1 \end{cases}$$

**52.** 
$$\begin{cases} x - 2y = 8 \\ 3x + 2y = -1 \end{cases}$$
 **53.** 
$$\begin{cases} 7x + 2y = 0 \\ 2x + y = -3 \end{cases}$$

54. 
$$\begin{cases} x + 2y + 2z = 5 \\ 2x + 4y + 7z = 19 \\ -2x - 5y - 2z = 8 \end{cases}$$

54. 
$$\begin{cases} x + 2y + 2z = 5 \\ 2x + 4y + 7z = 19 \\ -2x - 5y - 2z = 8 \end{cases}$$
 55. 
$$\begin{cases} 2x + y = -4 \\ y - 2z = 0 \\ 3x - 2z = -11 \end{cases}$$

**56.** Use the quadratic function  $y = ax^2 + bx + c$  to model the following data:

x (Age of a Driver)	y (Average Number of Automobile Accidents per Day in the United States)	
20	400	
40	150	
60	400	

Use Cramer's rule to determine values for a, b, and c. Then use the model to write a statement about the average number of automobile accidents in which 30-year-olds and 50-year-olds are involved daily.

# CHAPTER

# Chapter 9

In Exercises 1-2, solve each system of equations using matrices.

1. 
$$\begin{cases} x + 2y - z = -3 \\ 2x - 4y + z = -7 \\ -2x + 2y - 3z = 4 \end{cases}$$

2. 
$$\begin{cases} x - 2y + z = 2 \\ 2x - y - z = 1 \end{cases}$$

In Exercises 3-6, let

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}.$$

Carry out the indicated operations.

3. 
$$2B + 3C$$

5. 
$$C^{-1}$$

6. 
$$BC - 3B$$

7. If 
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \\ 1 & -1 & -2 \end{bmatrix}$$
 and  $B = \begin{bmatrix} -3 & 2 & 0 \\ 7 & -4 & 1 \\ -5 & 3 & -1 \end{bmatrix}$ , show that 9. Evaluate:

B is the inverse of A.

8. Consider the system

$$3x + 5y = 9$$

$$2x - 3y = -13$$
.

- **a.** Express the system in the form AX = B, where A, X, and B are appropriate matrices.
- **b.** Find  $A^{-1}$ , the inverse of the coefficient matrix.
- c. Use  $A^{-1}$  to solve the given system.

9. Evaluate: 
$$\begin{vmatrix} 4 & -1 & 3 \\ 0 & 5 & -1 \\ 5 & 2 & 4 \end{vmatrix}$$

10. Solve for x only using Cramer's rule:

$$\begin{cases} 3x + y - 2z = -3\\ 2x + 7y + 3z = 9\\ 4x - 3y - z = 7. \end{cases}$$

# Cumulative Review Exercises (Chapters 1-9)

Solve each equation or inequality in Exercises 1-6.

1. 
$$2x^2 = 4 - x$$

2. 
$$5x + 8 \le 7(1 + x)$$

3. 
$$\sqrt{2x+4} - \sqrt{x+3} - 1 = 0$$

**4.** 
$$3x^3 + 8x^2 - 15x + 4 = 0$$

5. 
$$e^{2x} - 14e^x + 45 = 0$$

6. 
$$\log_3 x + \log_3(x+2) = 1$$

7. Use matrices to solve this system:

$$\begin{cases} x - y + z = 17 \\ 2x + 3y + z = 8 \\ -4x + y + 5z = -2. \end{cases}$$

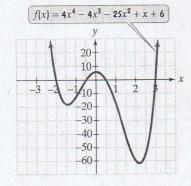
8. Solve for y using Cramer's rule:

$$\begin{cases} x - 2y + z = 7 \\ 2x + y - z = 0 \\ 3x + 2y - 2z = -2. \end{cases}$$

9. If 
$$f(x) = \sqrt{4x - 7}$$
, find  $f^{-1}(x)$ .

**10.** Graph: 
$$f(x) = \frac{x}{x^2 - 16}$$

11. Use the graph of  $f(x) = 4x^4 - 4x^3 - 25x^2 + x + 6$  shown in the figure to factor the polynomial completely.



- 12. Graph  $y = \log_2 x$  and  $y = \log_2(x + 1)$  in the same rectangular coordinate system.
- 13. Use the exponential decay model  $A = A_0 e^{kt}$  to solve this problem. A radioactive substance has a half-life of 40 days. There are initially 900 grams of the substance.
  - **a.** Find the decay model for this substance. Round *k* to the nearest thousandth.
  - **b.** How much of the substance will remain after 10 days? Round to the nearest hundredth of a gram.

**14.** Multiply the matrices: 
$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 2 & 0 \\ 1 & 1 \end{bmatrix}$$
.

15. Find the partial fraction decomposition of

$$\frac{3x^2 + 17x - 38}{(x-3)(x-2)(x+2)}.$$

In Exercises 16–19, graph each equation, function, or inequality in a rectangular coordinate system.

**16.** 
$$y = -\frac{2}{3}x - 1$$

17. 
$$3x - 5y < 15$$

**18.** 
$$f(x) = x^2 - 2x - 3$$

**19.** 
$$(x-1)^2 + (y+1)^2 = 9$$

20. Use synthetic division to divide  $x^3 - 6x + 4$  by x - 2

**21.** Graph: 
$$y = 2 \sin 2\pi x$$
,  $0 \le x \le 2$ .

22. Find the exact value of  $\cos\left[\tan^{-1}\left(-\frac{4}{3}\right)\right]$ .

23. Verify the identity:  $\frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x.$ 

**24.** Solve on the interval  $[0, 2\pi)$ :  $\cos^2 x + \sin x + 1 = 0$ .

25. If 
$$v = -6i + 5j$$
 and  $w = -7i + 3j$  find  $4w - 5v$ .