Exercise Set 9.2

Practice Exercises

In Exercises 1–24, use Gaussian elimination to find the complete solution to each system of equations, or show that none exists.

1. $\begin{cases} 5x + 12y + z = 10 \\ 2x + 5y + 2z = -1 \\ x + 2y - 3z = 5 \end{cases}$

2. $\begin{cases} 2x - 4y + z = 3 \\ x - 3y + z = 5 \\ 3x - 7y + 2z = 12 \end{cases}$

3. $\begin{cases} 5x + 8y - 6z = 14 \\ 3x + 4y - 2z = 8 \\ x + 2y - 2z = 3 \end{cases}$

4. $\begin{cases} 5x - 11y + 6z = 12 \\ -x + 3y - 2z = -4 \\ 3x - 5y + 2z = 4 \end{cases}$

5. $\begin{cases} 3x + 4y + 2z = 3 \\ 4x - 2y - 8z = -4 \\ x + y - z = 3 \end{cases}$

6. $\begin{cases} 2x - y - z = 0 \\ x + 2y + z = 3 \\ 3x + 4y + 2z = 8 \end{cases}$

7. $\begin{cases} 8x + 5y + 11z = 30 \\ -x - 4y + 2z = 3 \\ 2x - y + 5z = 12 \end{cases}$

8. $\begin{cases} x + y - 10z = -4 \\ x - 7z = -5 \\ 3x + 5y - 36z = -10 \end{cases}$

9. $\begin{cases} w - 2x - y - 3z = -9 \\ w + x - y = 0 \\ 3w + 4x + z = 6 \\ 2x - 2y + z = 3 \end{cases}$

10. $\begin{cases} 2w + x - 2y - z = 3 \\ w - 2x + y + z = 4 \\ -w - 8x + 7y + 5z = 13 \\ 3w + x - 2y + 2z = 6 \end{cases}$

11. $\begin{cases} 2w + x - y = 3 \\ w - 3x + 2y = -4 \\ 3w + x - 3y + z = 1 \\ w + 2x - 4y - z = -2 \end{cases}$

12. $\begin{cases} 2w - x + 3y + z = 0 \\ 3w + 2x + 4y - z = 0 \\ 5w - 2x - 2y - z = 0 \\ 2w + 3x - 7y - 5z = 0 \end{cases}$

13. $\begin{cases} w - 3x + y - 4z = 4 \\ -2w + x + 2y = -2 \\ 3w - 2x + y - 6z = 2 \\ -w + 3x + 2y - z = -6 \end{cases}$

14. $\begin{cases} 3w + 2x - y + 2z = -12 \\ 4w - x + y + 2z = 1 \\ w + x + y + z = -2 \\ -2w + 3x + 2y - 3z = 10 \end{cases}$

15. $\begin{cases} 2x + y - z = 2 \\ 3x + 3y - 2z = 3 \end{cases}$

16. $\begin{cases} 3x + 2y - z = 5 \\ x + 2y - z = 1 \end{cases}$

17. $\begin{cases} x + 2y + 3z = 5 \\ y - 5z = 0 \end{cases}$

18. $\begin{cases} 3x - y + 4z = 8 \\ y + 2z = 1 \end{cases}$

19. $\begin{cases} x + y - 2z = 2 \\ 3x - y - 6z = -7 \end{cases}$

20. $\begin{cases} -2x - 5y + 10z = 19 \\ x + 2y - 4z = 12 \end{cases}$

21. $\begin{cases} w + x - y + z = -2 \\ 2w - x + 2y - z = 7 \\ -w + 2x + y + 2z = -1 \end{cases}$

22. $\begin{cases} 2w - 3x + 4y + z = 7 \\ w - x + 3y - 5z = 10 \\ 3w + x - 2y - 2z = 6 \end{cases}$

23. $\begin{cases} w + 2x + 3y - z = 7 \\ 2x - 3y + z = 4 \\ w - 4x + y = 3 \end{cases}$

24. $\begin{cases} w - x + z = 0 \\ w - 4x + y + 2z = 0 \\ 3w - y + 2z = 0 \end{cases}$

Practice Plus

In Exercises 25–28, the first screen shows the augmented matrix, A, for a nonsquare linear system of three equations in four variables, w, x, y, and z. The second screen shows the reduced row-echelon form of matrix A. For each exercise,

a. *Write the system represented by A.*

b. *Use the reduced row-echelon form of A to find the system's complete solution.*

25.

```
[A]
[[4  -2  2   -3 0]
 [7  -1  -1  -3 0]
 [1  1   1   -1 0]]
```

```
rref([A])
 [[1 0 0 -.5 0]
  [0 1 0 0    0]
  [0 0 1 -.5 0]]
```

26.

```
[A]
[[2 17  -23 40 0]
 [2 5   1   3  0]
 [0 1   -2  3  0]]
```

```
rref([A])
 [[1 0 5.5 0 0]
  [0 1 -2  0 0]
  [0 0 0   1 0]]
```

27.

```
[A]
[[1 2  5  5 -3]
 [1 1  3  4 -1]
 [1 -1 -1 2 3 ]]
```

```
rref([A])
 [[1 0 1 3 1 ]
  [0 1 2 1 -2]
  [0 0 0 0 0 ]]
```

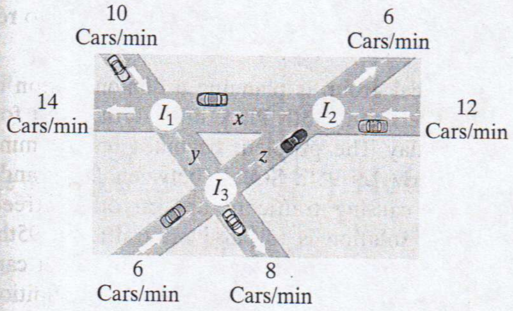
28.

```
[A]
  [[1  0   1  1  0]
   [1  -1  2  3  0]
   [3  -2  5  7  0]]
```

```
rref([A])
  [[1  0  1   1  0]
   [0  1  -1  -2 0]
   [0  0  0   0  0]]
```

Application Exercises

The figure for Exercises 29–32 shows the intersections of three one-way streets. To keep traffic moving, the number of cars per minute entering an intersection must equal the number exiting that intersection. For intersection I_1, $x + 10$ cars enter and $y + 14$ cars exit per minute. Thus, $x + 10 = y + 14$.



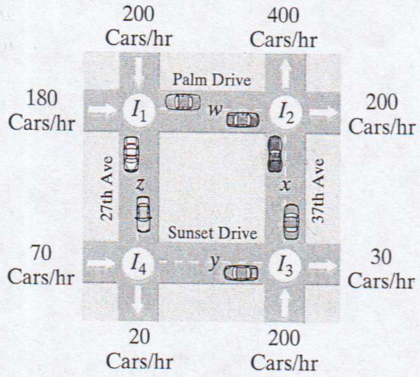
29. Write an equation for intersection I_2 that keeps traffic moving.

30. Write an equation for intersection I_3 that keeps traffic moving.

31. Use Gaussian elimination to solve the system formed by the equation given prior to Exercise 29 and the two equations that you obtained in Exercises 29–30.

32. Use your ordered solution obtained in Exercise 31 to solve this exercise. If construction limits z to 4 cars per minute, how many cars per minute must pass between the other intersections to keep traffic flowing?

33. The figure shows the intersections of four one-way streets.



a. Set up a system of equations that keeps traffic moving.

b. Use Gaussian elimination to solve the system.

c. If construction limits z to 50 cars per hour, how many cars per hour must pass between the other intersections to keep traffic moving?

34. The vitamin content per ounce for three foods is given in the following table.

	Milligrams per Ounce		
	Thiamin	**Riboflavin**	**Niacin**
Food A	3	7	1
Food B	1	5	3
Food C	3	8	2

a. Use matrices to show that no combination of these foods can provide exactly 14 mg of thiamin, 32 mg of riboflavin, and 9 mg of niacin.

b. Use matrices to describe in practical terms what happens if the riboflavin requirement is increased by 5 mg and the other requirements stay the same.

35. Three foods have the following nutritional content per ounce.

	Units per Ounce		
	Vitamin A	**Iron**	**Calcium**
Food 1	20	20	10
Food 2	30	10	10
Food 3	10	10	30

a. A diet must consist precisely of 220 units of vitamin A, 180 units of iron, and 340 units of calcium. However, the dietician runs out of Food 1. Use a matrix approach to show that under these conditions the dietary requirements cannot be met.

b. Now suppose that all three foods are available. Use matrices to give two possible ways to meet the iron and calcium requirements with the three foods.

36. A company that manufactures products A, B, and C does both manufacturing and testing. The hours needed to manufacture and test each product are shown in the table.

	Hours Needed Weekly to Manufacture	**Hours Needed Weekly to Test**
Product *A*	7	2
Product *B*	6	2
Product *C*	3	1

The company has exactly 67 hours per week available for manufacturing and 20 hours per week available for testing. Give two different combinations for the number of products that can be manufactured and tested weekly.

Writing in Mathematics

37. Describe what happens when Gaussian elimination is used to solve an inconsistent system.

38. Describe what happens when Gaussian elimination is used to solve a system with dependent equations.