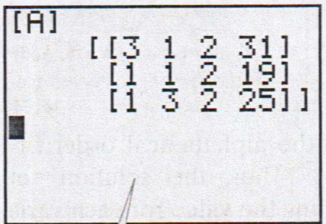
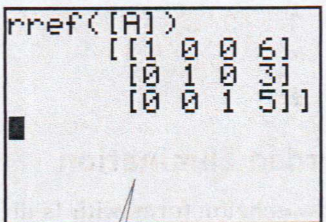
Technology

Most graphing utilities can convert a matrix to reduced row-echelon form. Enter the system's augmented matrix and name it A. Then use the RREF (reduced row-echelon form) command on matrix A.



This is the augmented matrix for the system in Example 4.



This is the matrix in reduced row-echelon form we obtained in Example 4.

EXAMPLE 4 Using Gauss-Jordan Elimination

Use Gauss-Jordan elimination to solve the system:

$$\begin{cases} 3x + y + 2z = 31 \\ x + y + 2z = 19 \\ x + 3y + 2z = 25. \end{cases}$$

Solution In Example 2, we used Gaussian elimination to obtain the following matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 19 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 1 & 5 \end{array}\right].$$

We want 0s in these positions.

To use Gauss-Jordan elimination, we need 0s both above and below the 1s in the main diagonal. We use the 1 in the second row, second column to get a 0 above it.

Replace row 1 in the previous matrix by $-1R_2 + R_1$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

We want 0s in these positions.

We use the 1 in the third column to get 0s above it.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Replace row 2 in the previous matrix by $-2R_3 + R_2$.

This last matrix corresponds to

$$x = 6, \quad y = 3, \quad z = 5.$$

As we found in Example 2, the solution set is $\{(6, 3, 5)\}$. ■

Check Point 4 Solve the system in Check Point 2 using Gauss-Jordan elimination. Begin by working with the matrix that you obtained in Check Point 2.

Exercise Set 9.1

Practice Exercises

In Exercises 1–8, write the augmented matrix for each system of linear equations.

1. $\begin{cases} 2x + y + 2z = 2 \\ 3x - 5y - z = 4 \\ x - 2y - 3z = -6 \end{cases}$

2. $\begin{cases} 3x - 2y + 5z = 31 \\ x + 3y - 3z = -12 \\ -2x - 5y + 3z = 11 \end{cases}$

3. $\begin{cases} x - y + z = 8 \\ y - 12z = -15 \\ z = 1 \end{cases}$

4. $\begin{cases} x - 2y + 3z = 9 \\ y + 3z = 5 \\ z = 2 \end{cases}$

5. $\begin{cases} 5x - 2y - 3z = 0 \\ x + y = 5 \\ 2x - 3z = 4 \end{cases}$

6. $\begin{cases} x - 2y + z = 10 \\ 3x + y = 5 \\ 7x + 2z = 2 \end{cases}$

7. $\begin{cases} 2w + 5x - 3y + z = 2 \\ 3x + y = 4 \\ w - x + 5y = 9 \\ 5w - 5x - 2y = 1 \end{cases}$

8. $\begin{cases} 4w + 7x - 8y + z = 3 \\ 5x + y = 5 \\ w - x - y = 17 \\ 2w - 2x + 11y = 4 \end{cases}$

In Exercises 9–12, write the system of linear equations represented by the augmented matrix. Use x, y, and z, or, if necessary, w, x, y, and z, for the variables.

9. $\left[\begin{array}{ccc|c} 5 & 0 & 3 & -11 \\ 0 & 1 & -4 & 12 \\ 7 & 2 & 0 & 3 \end{array}\right]$

10. $\left[\begin{array}{ccc|c} 7 & 0 & 4 & -13 \\ 0 & 1 & -5 & 11 \\ 2 & 7 & 0 & 6 \end{array}\right]$

11. $\left[\begin{array}{cccc|c} 1 & 1 & 4 & 1 & 3 \\ -1 & 1 & -1 & 0 & 7 \\ 2 & 0 & 0 & 5 & 11 \\ 0 & 0 & 12 & 4 & 5 \end{array}\right]$

12. $\left[\begin{array}{cccc|c} 4 & 1 & 5 & 1 & 6 \\ 1 & -1 & 0 & -1 & 8 \\ 3 & 0 & 0 & 7 & 4 \\ 0 & 0 & 11 & 5 & 3 \end{array}\right]$

In Exercises 13–18, perform each matrix row operation and write the new matrix.

13. $\left[\begin{array}{ccc|c} 2 & -6 & 4 & 10 \\ 1 & 5 & -5 & 0 \\ 3 & 0 & 4 & 7 \end{array}\right] \quad \frac{1}{2}R_1$

14. $\left[\begin{array}{ccc|c} 3 & -12 & 6 & 9 \\ 1 & -4 & 4 & 0 \\ 2 & 0 & 7 & 4 \end{array}\right]$ $\frac{1}{3}R_1$

15. $\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 3 & 1 & -1 & 7 \\ 2 & -2 & 1 & 3 \end{array}\right]$ $-3R_1 + R_2$

16. $\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{array}\right]$ $-3R_1 + R_2$

17. $\left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 3 \\ 0 & 1 & -2 & -1 & 0 \\ 2 & 0 & 3 & 4 & 11 \\ 5 & 1 & 2 & 4 & 6 \end{array}\right]$ $\begin{array}{l} \\ \\ -2R_1 + R_3 \\ -5R_1 + R_4 \end{array}$

18. $\left[\begin{array}{cccc|c} 1 & -5 & 2 & -2 & 4 \\ 0 & 1 & -3 & -1 & 0 \\ 3 & 0 & 2 & -1 & 6 \\ -4 & 1 & 4 & 2 & -3 \end{array}\right]$ $\begin{array}{l} \\ \\ -3R_1 + R_3 \\ 4R_1 + R_4 \end{array}$

In Exercises 19–20, a few steps in the process of simplifying the given matrix to row-echelon form, with 1s down the diagonal from upper left to lower right, and 0s below the 1s, are shown. Fill in the missing numbers in the steps that are shown.

19. $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 2 & 3 & -1 & -2 \\ 3 & -2 & -9 & 9 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 5 & \square & \square \\ 0 & 1 & \square & \square \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 1 & \square & \square \\ 0 & 1 & \square & \square \end{array}\right]$

20. $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ -3 & 4 & -1 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 5 & \square & \square \\ 0 & -2 & \square & \square \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & \square & \square \\ 0 & -2 & \square & \square \end{array}\right]$

In Exercises 21–38, solve each system of equations using matrices. Use Gaussian elimination with back-substitution or Gauss-Jordan elimination.

21. $\begin{cases} x + y - z = -2 \\ 2x - y + z = 5 \\ -x + 2y + 2z = 1 \end{cases}$

22. $\begin{cases} x - 2y - z = 2 \\ 2x - y + z = 4 \\ -x + y - 2z = -4 \end{cases}$

23. $\begin{cases} x + 3y = 0 \\ x + y + z = 1 \\ 3x - y - z = 11 \end{cases}$

24. $\begin{cases} 3y - z = -1 \\ x + 5y - z = -4 \\ -3x + 6y + 2z = 11 \end{cases}$

25. $\begin{cases} 2x - y - z = 4 \\ x + y - 5z = -4 \\ x - 2y = 4 \end{cases}$

26. $\begin{cases} x - 3z = -2 \\ 2x + 2y + z = 4 \\ 3x + y - 2z = 5 \end{cases}$

27. $\begin{cases} x + y + z = 4 \\ x - y - z = 0 \\ x - y + z = 2 \end{cases}$

28. $\begin{cases} 3x + y - z = 0 \\ x + y + 2z = 6 \\ 2x + 2y + 3z = 10 \end{cases}$

29. $\begin{cases} x + 2y = z - 1 \\ x = 4 + y - z \\ x + y - 3z = -2 \end{cases}$

30. $\begin{cases} 2x + y = z + 1 \\ 2x = 1 + 3y - z \\ x + y + z = 4 \end{cases}$

31. $\begin{cases} 3a - b - 4c = 3 \\ 2a - b + 2c = -8 \\ a + 2b - 3c = 9 \end{cases}$

32. $\begin{cases} 3a + b - c = 0 \\ 2a + 3b - 5c = 1 \\ a - 2b + 3c = -4 \end{cases}$

33. $\begin{cases} 2x + 2y + 7z = -1 \\ 2x + y + 2z = 2 \\ 4x + 6y + z = 15 \end{cases}$

34. $\begin{cases} 3x + 2y + 3z = 3 \\ 4x - 5y + 7z = 1 \\ 2x + 3y - 2z = 6 \end{cases}$

35. $\begin{cases} w + x + y + z = 4 \\ 2w + x - 2y - z = 0 \\ w - 2x - y - 2z = -2 \\ 3w + 2x + y + 3z = 4 \end{cases}$

36. $\begin{cases} w + x + y + z = 5 \\ w + 2x - y - 2z = -1 \\ w - 3x - 3y - z = -1 \\ 2w - x + 2y - z = -2 \end{cases}$

37. $\begin{cases} 3w - 4x + y + z = 9 \\ w + x - y - z = 0 \\ 2w + x + 4y - 2z = 3 \\ -w + 2x + y - 3z = 3 \end{cases}$

38. $\begin{cases} 2w + y - 3z = 8 \\ w - x + 4z = -10 \\ 3w + 5x - y - z = 20 \\ w + x - y - z = 6 \end{cases}$

Practice Plus

39. Find the quadratic function $f(x) = ax^2 + bx + c$ for which $f(-2) = -4, f(1) = 2$, and $f(2) = 0$.

40. Find the quadratic function $f(x) = ax^2 + bx + c$ for which $f(-1) = 5, f(1) = 3$, and $f(2) = 5$.

41. Find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ for which $f(-1) = 0, f(1) = 2, f(2) = 3$, and $f(3) = 12$.

42. Find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ for which $f(-1) = 3, f(1) = 1, f(2) = 6$, and $f(3) = 7$.

43. Solve the system:

$$\begin{cases} 2\ln w + \ln x + 3\ln y - 2\ln z = -6 \\ 4\ln w + 3\ln x + \ln y - \ln z = -2 \\ \ln w + \ln x + \ln y + \ln z = -5 \\ \ln w + \ln x - \ln y - \ln z = 5. \end{cases}$$

(*Hint:* Let $A = \ln w, B = \ln x, C = \ln y$, and $D = \ln z$. Solve the system for A, B, C, and D. Then use the logarithmic equations to find w, x, y, and z.)

44. Solve the system:

$$\begin{cases} \ln w + \ln x + \ln y + \ln z = -1 \\ -\ln w + 4\ln x + \ln y - \ln z = 0 \\ \ln w - 2\ln x + \ln y - 2\ln z = 11 \\ -\ln w - 2\ln x + \ln y + 2\ln z = -3. \end{cases}$$

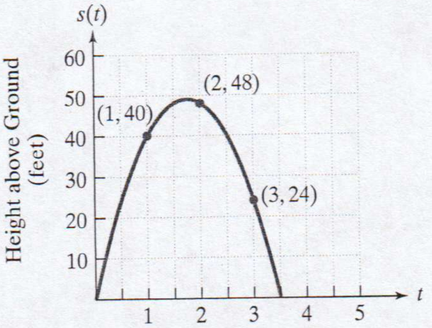
(*Hint:* Let $A = \ln w, B = \ln x, C = \ln y$, and $D = \ln z$. Solve the system for A, B, C, and D. Then use the logarithmic equations to find w, x, y, and z.)

Application Exercises

45. A ball is thrown straight upward. A position function

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0$$

can be used to describe the ball's height, $s(t)$, in feet, after t seconds.



a. Use the points labeled in the graph to find the values of a, v_0, and s_0. Solve the system of linear equations involving a, v_0, and s_0 using matrices.

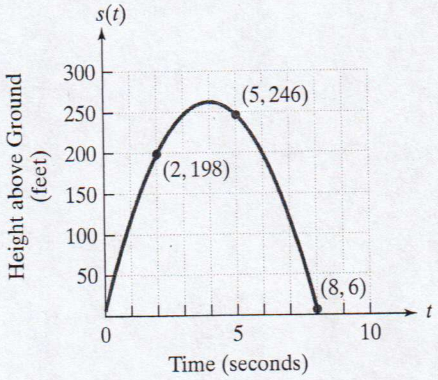
b. Find and interpret $s(3.5)$. Identify your solution as a point on the graph shown.

c. After how many seconds does the ball reach its maximum height? What is its maximum height?

46. A football is kicked straight upward. A position function

$$s(t) = \frac{1}{2}at^2 + v_0t + s_0$$

can be used to describe the ball's height, $s(t)$, in feet, after t seconds.



a. Use the points labeled in the graph to find the values of a, v_0, and s_0. Solve the system of linear equations involving a, v_0, and s_0 using matrices.

b. Find and interpret $s(7)$. Identify your solution as a point on the graph shown.

c. After how many seconds does the ball reach its maximum height? What is its maximum height?

Write a system of linear equations in three or four variables to solve Exercises 47–50. Then use matrices to solve the system.

47. Three foods have the following nutritional content per ounce.

	Calories	Protein (in grams)	Vitamin C (in milligrams)
Food *A*	40	5	30
Food *B*	200	2	10
Food *C*	400	4	300

If a meal consisting of the three foods allows exactly 660 calories, 25 grams of protein, and 425 milligrams of vitamin C, how many ounces of each kind of food should be used?

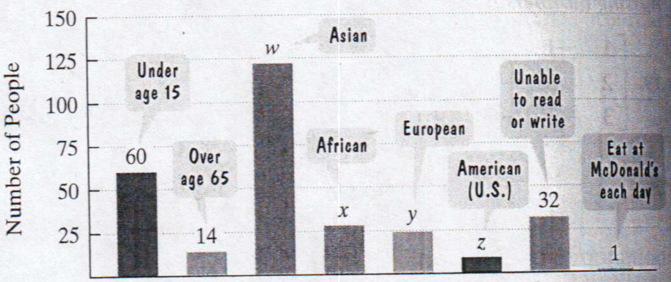
48. A furniture company produces three types of desks: a children's model, an office model, and a deluxe model. Each desk is manufactured in three stages: cutting, construction, and finishing. The time requirements for each model and manufacturing stage are given in the following table.

	Children's model	Office model	Deluxe model
Cutting	2 hr	3 hr	2 hr
Construction	2 hr	1 hr	3 hr
Finishing	1 hr	1 hr	2 hr

Each week the company has available a maximum of 100 hours for cutting, 100 hours for construction, and 65 hours for finishing. If all available time must be used, how many of each type of desk should be produced each week?

49. Imagine the entire global population as a village of precisely 200 people. The bar graph shows some numeric observations based on this scenario.

Earth's Population as a Village of 200 People

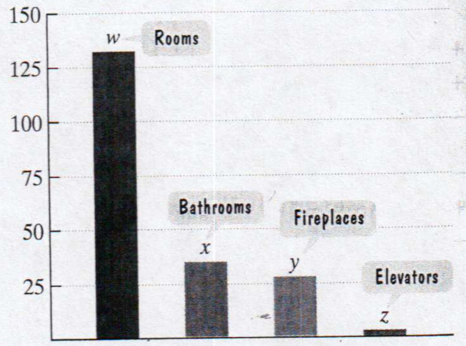


Source: Gary Rimmer, *Number Freaking*, The Disinformation Company Ltd., 2006

Combined, there are 183 Asians, Africans, Europeans, and Americans in the village. The number of Asians exceeds the number of Africans and Europeans by 70. The difference between the number of Europeans and Americans is 15. If the number of Africans is doubled, their population exceeds the number of Europeans and Americans by 23. Determine the number of Asians, Africans, Europeans, and Americans in the global village.

50. The bar graph shows the number of rooms, bathrooms, fireplaces, and elevators in the U.S. White House.

The U.S. White House by the Numbers



Source: The White House