Solution We need to find values for a, b, and c in $y = ax^2 + bx + c$. We can do so by solving a system of three linear equations in a, b, and c. We obtain the three equations by using the values of x and y from the data as follows:

$y = ax^2 + bx + c$ — *Use the quadratic function to model the data.*

When $x = 4, y = 1682$:
When $x = 7, y = 626$:
When $x = 9, y = 967$:

$$\begin{cases} 1682 = a \cdot 4^2 + b \cdot 4 + c \\ 626 = a \cdot 7^2 + b \cdot 7 + c \\ 967 = a \cdot 9^2 + b \cdot 9 + c \end{cases} \quad \text{or} \quad \begin{cases} 16a + 4b + c = 1682 \\ 49a + 7b + c = 626 \\ 81a + 9b + c = 967. \end{cases}$$

The easiest way to solve this system is to eliminate c from two pairs of equations, obtaining two equations in a and b. Solving this [illegible] and $c = 6016$. We now substitute the values f[illegible] The function that models the given data is

$$y = 104.5x^2 - 150 \text{[illegible]}$$

We can use the model that we obtained [illegible] males who average, say, 6 hours of sleep. First [illegible]

$$f(x) = 104.5x^2 - 15 \text{[illegible]}$$

Substitute 6 for x:

$$f(6) = 104.5(6)^2 - 1501.5 \text{[illegible]}$$

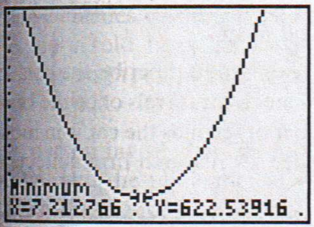
According to the model, the death rate for ma[illegible] deaths per 100,000 males.

Technology

The graph of

$y = 104.5x^2 - 1501.5x + 6016$

is displayed in a [3, 12, 1] by [500, 2000, 100] viewing rectangle. The minimum function feature shows that the lowest point on the graph, the vertex, is approximately (7.2, 622.5). Men who average 7.2 hours of sleep are in the group with the lowest death rate, approximately 622.5 deaths per 100,000 males.



Check Point 4 Find the quadratic function $y = ax^2 + bx + c$ whose graph passes through the points (1, 4), (2, 1), and (3, 4).

Exercise Set 8.2

Practice Exercises

In Exercises 1–4, determine if the given ordered triple is a solution of the system.

1. (2, −1, 3)
$$\begin{cases} x + y + z = 4 \\ x - 2y - z = 1 \\ 2x - y - 2z = -1 \end{cases}$$

2. (5, −3, −2)
$$\begin{cases} x + y + z = 0 \\ x + 2y - 3z = 5 \\ 3x + 4y + 2z = -1 \end{cases}$$

3. (4, 1, 2)
$$\begin{cases} x - 2y = 2 \\ 2x + 3y = 11 \\ y - 4z = -7 \end{cases}$$

4. (−1, 3, 2)
$$\begin{cases} x - 2z = -5 \\ y - 3z = -3 \\ 2x - z = -4 \end{cases}$$

Solve each system in Exercises 5–18.

5. $$\begin{cases} x + y + 2z = 11 \\ x + y + 3z = 14 \\ x + 2y - z = 5 \end{cases}$$

6. $$\begin{cases} 2x + y - 2z = -1 \\ 3x - 3y - z = 5 \\ x - 2y + 3z = 6 \end{cases}$$

7. $$\begin{cases} 4x - y + 2z = 11 \\ x + 2y - z = -1 \\ 2x + 2y - 3z = -1 \end{cases}$$

8. $$\begin{cases} x - y + 3z = 8 \\ 3x + y - 2z = -2 \\ 2x + 4y + z = 0 \end{cases}$$

9. $$\begin{cases} 3x + 2y - 3z = -2 \\ 2x - 5y + 2z = -2 \\ 4x - 3y + 4z = 10 \end{cases}$$

10. $$\begin{cases} 2x + 3y + 7z = 13 \\ 3x + 2y - 5z = -22 \\ 5x + 7y - 3z = -28 \end{cases}$$

11. $$\begin{cases} 2x - 4y + 3z = 17 \\ x + 2y - z = 0 \\ 4x - y - z = 6 \end{cases}$$

12. $$\begin{cases} x + z = 3 \\ x + 2y - z = 1 \\ 2x - y + z = 3 \end{cases}$$

13. $$\begin{cases} 2x + y = 2 \\ x + y - z = 4 \\ 3x + 2y + z = 0 \end{cases}$$

14. $$\begin{cases} x + 3y + 5z = 20 \\ y - 4z = -16 \\ 3x - 2y + 9z = 36 \end{cases}$$

15. $$\begin{cases} x + y = -4 \\ y - z = 1 \\ 2x + y + 3z = -21 \end{cases}$$

16. $$\begin{cases} x + y = 4 \\ x + z = 4 \\ y + z = 4 \end{cases}$$

17. $$\begin{cases} 3(2x + y) + 5z = -1 \\ 2(x - 3y + 4z) = -9 \\ 4(1 + x) = -3(z - 3y) \end{cases}$$

18. $$\begin{cases} 7z - 3 = 2(x - 3y) \\ 5y + 3z - 7 = 4x \\ 4 + 5z = 3(2x - y) \end{cases}$$

$\overset{9}{\cancel{18}} \cdot \frac{7}{\underset{1}{\cancel{2}}} = \overset{2}{\cancel{18}} \cdot \frac{1}{\underset{1}{\cancel{9}}}x + \overset{2}{\cancel{18}} \cdot \frac{26}{\underset{1}{\cancel{9}}}$	Divide out common factors in the multiplications.
$63 = 2x + 52$	Complete the multiplications. The fractions are now cleared.
$63 - 52 = 2x + 52 - 52$	Subtract 52 from both sides to get constants on the left.
$11 = 2x$	Simplify.
$\frac{11}{2} = \frac{2x}{2}$	Divide both sides by 2.
$\frac{11}{2} = x$	Simplify.

The formula indicates that if the high-humor group averages a level of depression of 3.5 in response to a negative life event, the intensity of that event is $\frac{11}{2}$, or 5.5. This is illustrated on the line graph for the high-humor group in **Figure 1.14**.

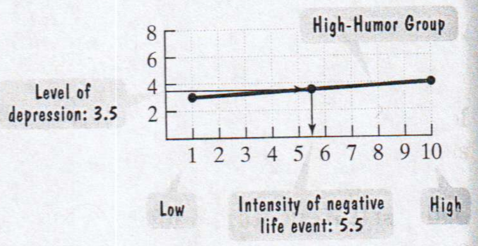


Figure 1.14

Check Point 8 Use the model for the low-humor group given in Example 8 to solve this problem. If the low-humor group averages a level of depression of 10 in response to a negative life event, what is the intensity of that event? How is the solution shown on the blue line graph in **Figure 1.13**?

Exercise Set 1.2

Practice Exercises

In Exercises 1–16, solve and check each linear equation.

1. $7x - 5 = 72$

2. $6x - 3 = 63$

3. $11x - (6x - 5) = 40$

4. $5x - (2x - 10) = 35$

5. $2x - 7 = 6 + x$

6. $3x + 5 = 2x + 13$

7. $7x + 4 = x + 16$

8. $13x + 14 = 12x - 5$

9. $3(x - 2) + 7 = 2(x + 5)$

10. $2(x - 1) + 3 = x - 3(x + 1)$

11. $3(x - 4) - 4(x - 3) = x + 3 - (x - 2)$

12. $2 - (7x + 5) = 13 - 3x$

13. $16 = 3(x - 1) - (x - 7)$

14. $5x - (2x + 2) = x + (3x - 5)$

15. $25 - [2 + 5y - 3(y + 2)] = -3(2y - 5) - [5(y - 1) - 3y + 3]$

16. $45 - [4 - 2y - 4(y + 7)] = -4(1 + 3y) - [4 - 3(y + 2) - 2(2y - 5)]$

Exercises 17–30 contain linear equations with constants in denominators. Solve each equation.

17. $\frac{x}{3} = \frac{x}{2} - 2$

18. $\frac{x}{5} = \frac{x}{6} + 1$

19. $20 - \frac{x}{3} = \frac{x}{2}$

20. $\frac{x}{5} - \frac{1}{2} = \frac{x}{6}$

21. $\frac{3x}{5} = \frac{2x}{3} + 1$

22. $\frac{x}{2} = \frac{3x}{4} + 5$

23. $\frac{3x}{5} - x = \frac{x}{10} - \frac{5}{2}$

24. $2x - \frac{2x}{7} = \frac{x}{2} + \frac{17}{2}$

25. $\frac{x + 3}{6} = \frac{3}{8} + \frac{x - 5}{4}$

26. $\frac{x + 1}{4} = \frac{1}{6} + \frac{2 - x}{3}$

27. $\frac{x}{4} = 2 + \frac{x - 3}{3}$

28. $5 + \frac{x - 2}{3} = \frac{x + 3}{8}$.

29. $\frac{x + 1}{3} = 5 - \frac{x + 2}{7}$

30. $\frac{3x}{5} - \frac{x - 3}{2} = \frac{x + 2}{3}$

Exercises 31–50 contain rational equations with variables in denominators. For each equation, **a.** *Write the value or values of the variable that make a denominator zero. These are the restrictions on the variable.* **b.** *Keeping the restrictions in mind, solve the equation.*

31. $\frac{4}{x} = \frac{5}{2x} + 3$

32. $\frac{5}{x} = \frac{10}{3x} + 4$

33. $\frac{2}{x} + 3 = \frac{5}{2x} + \frac{13}{4}$

34. $\frac{7}{2x} - \frac{5}{3x} = \frac{22}{3}$

35. $\frac{2}{3x} + \frac{1}{4} = \frac{11}{6x} - \frac{1}{3}$

36. $\frac{5}{2x} - \frac{8}{9} = \frac{1}{18} - \frac{1}{3x}$

37. $\frac{x - 2}{2x} + 1 = \frac{x + 1}{x}$

38. $\frac{4}{x} = \frac{9}{5} - \frac{7x - 4}{5x}$

39. $\frac{1}{x - 1} + 5 = \frac{11}{x - 1}$

40. $\frac{3}{x + 4} - 7 = \frac{-4}{x + 4}$

41. $\frac{8x}{x + 1} = 4 - \frac{8}{x + 1}$

42. $\frac{2}{x - 2} = \frac{x}{x - 2} - 2$

43. $\frac{3}{2x - 2} + \frac{1}{2} = \frac{2}{x - 1}$

44. $\dfrac{3}{x+3} = \dfrac{5}{2x+6} + \dfrac{1}{x-2}$

45. $\dfrac{3}{x+2} + \dfrac{2}{x-2} = \dfrac{8}{(x+2)(x-2)}$

46. $\dfrac{5}{x+2} + \dfrac{3}{x-2} = \dfrac{12}{(x+2)(x-2)}$

47. $\dfrac{2}{x+1} - \dfrac{1}{x-1} = \dfrac{2x}{x^2-1}$

48. $\dfrac{4}{x+5} + \dfrac{2}{x-5} = \dfrac{32}{x^2-25}$

49. $\dfrac{1}{x-4} - \dfrac{5}{x+2} = \dfrac{6}{x^2-2x-8}$

50. $\dfrac{6}{x+3} - \dfrac{5}{x-2} = \dfrac{-20}{x^2+x-6}$

In Exercises 51–56, find all values of x satisfying the given conditions.

51. $y_1 = 5(2x-8) - 2$, $y_2 = 5(x-3) + 3$, and $y_1 = y_2$.

52. $y_1 = 7(3x-2) + 5$, $y_2 = 6(2x-1) + 24$, and $y_1 = y_2$.

53. $y_1 = \dfrac{x-3}{5}$, $y_2 = \dfrac{x-5}{4}$, and $y_1 - y_2 = 1$.

54. $y_1 = \dfrac{x+1}{4}$, $y_2 = \dfrac{x-2}{3}$, and $y_1 - y_2 = -4$.

55. $y_1 = \dfrac{5}{x+4}$, $y_2 = \dfrac{3}{x+3}$, $y_3 = \dfrac{12x+19}{x^2+7x+12}$, and $y_1 + y_2 = y_3$.

56. $y_1 = \dfrac{2x-1}{x^2+2x-8}$, $y_2 = \dfrac{2}{x+4}$, $y_3 = \dfrac{1}{x-2}$, and $y_1 + y_2 = y_3$.

In Exercises 57–60, find all values of x such that y = 0.

57. $y = 4[x - (3 - x)] - 7(x + 1)$

58. $y = 2[3x - (4x - 6)] - 5(x - 6)$

59. $y = \dfrac{x+6}{3x-12} - \dfrac{5}{x-4} - \dfrac{2}{3}$

60. $y = \dfrac{1}{5x+5} - \dfrac{3}{x+1} + \dfrac{7}{5}$

In Exercises 61–68, determine whether each equation is an identity, a conditional equation, or an inconsistent equation.

61. $4(x - 7) = 4x - 28$

62. $4(x - 7) = 4x + 28$

63. $2x + 3 = 2x - 3$

64. $\dfrac{7x}{x} = 7$

65. $4x + 5x = 8x$

66. $8x + 2x = 9x$

67. $\dfrac{2x}{x-3} = \dfrac{6}{x-3} + 4$

68. $\dfrac{3}{x-3} = \dfrac{x}{x-3} + 3$

The equations in Exercises 69–80 combine the types of equations we have discussed in this section. Solve each equation. Then state whether the equation is an identity, a conditional equation, or an inconsistent equation.

69. $\dfrac{x+5}{2} - 4 = \dfrac{2x-1}{3}$

70. $\dfrac{x+2}{7} = 5 - \dfrac{x+1}{3}$

71. $\dfrac{2}{x-2} = 3 + \dfrac{x}{x-2}$

72. $\dfrac{6}{x+3} + 2 = \dfrac{-2x}{x+3}$

73. $8x - (3x + 2) + 10 = 3x$

74. $2(x + 2) + 2x = 4(x + 1)$

75. $\dfrac{2}{x} + \dfrac{1}{2} = \dfrac{3}{4}$

76. $\dfrac{3}{x} - \dfrac{1}{6} = \dfrac{1}{3}$

77. $\dfrac{4}{x-2} + \dfrac{3}{x+5} = \dfrac{7}{(x+5)(x-2)}$

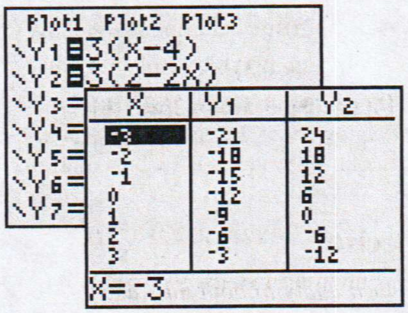
78. $\dfrac{1}{x-1} = \dfrac{1}{(2x+3)(x-1)} + \dfrac{4}{2x+3}$

79. $\dfrac{4x}{x+3} - \dfrac{12}{x-3} = \dfrac{4x^2+36}{x^2-9}$

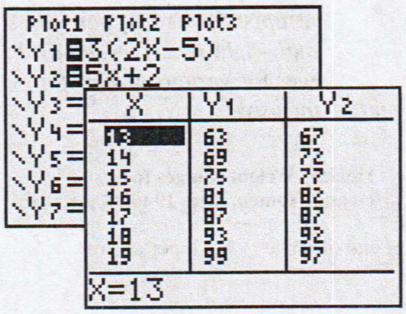
80. $\dfrac{4}{x^2+3x-10} - \dfrac{1}{x^2+x-6} = \dfrac{3}{x^2-x-12}$

In Exercises 81–84, use the [Y=] *screen to write the equation being solved. Then use the table to solve the equation.*

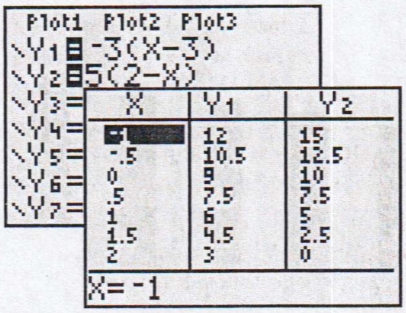
81.



82.



83.



84.