

EXAMPLE 7 An Application of the Law of Sines

Two fire-lookout stations are 20 miles apart, with station B directly east of station A. Both stations spot a fire on a mountain to the north. The bearing from station A to the fire is N50°E (50° east of north). The bearing from station B to the fire is N36°W (36° west of north). How far, to the nearest tenth of a mile, is the fire from station A?

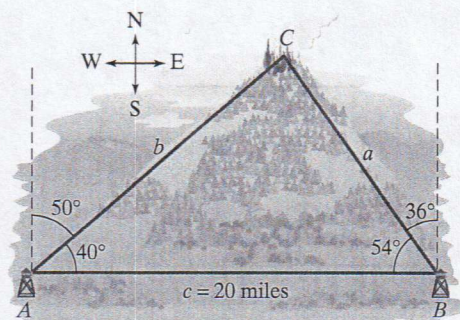


Figure 7.12

Solution Figure 7.12 shows the information given in the problem. The distance from station A to the fire is represented by b . Notice that the angles describing the bearing from each station to the fire, 50° and 36°, are not interior angles of triangle ABC . Using a north-south line, the interior angles are found as follows:

$$A = 90^\circ - 50^\circ = 40^\circ \quad B = 90^\circ - 36^\circ = 54^\circ.$$

To find b using the Law of Sines, we need a known side and an angle opposite that side. Because $c = 20$ miles, we find angle C using a 180° angle sum in the triangle. Thus,

$$C = 180^\circ - A - B = 180^\circ - 40^\circ - 54^\circ = 86^\circ.$$

The ratio $\frac{c}{\sin C}$, or $\frac{20}{\sin 86^\circ}$, is now known. We use this ratio and the Law of Sines to find b .

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the Law of Sines.

$$\frac{b}{\sin 54^\circ} = \frac{20}{\sin 86^\circ}$$

$$c = 20, B = 54^\circ, \text{ and } C = 86^\circ.$$

$$b = \frac{20 \sin 54^\circ}{\sin 86^\circ} \approx 16.2 \quad \text{Multiply both sides by } \sin 54^\circ \text{ and solve for } b.$$

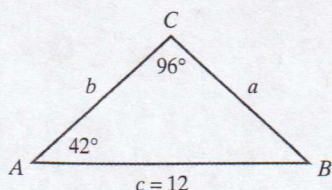
The fire is approximately 16.2 miles from station A.

Check Point 7 Two fire-lookout stations are 13 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N35°E and the bearing of the fire from station B is N49°W. How far, to the nearest tenth of a mile, is the fire from station B?

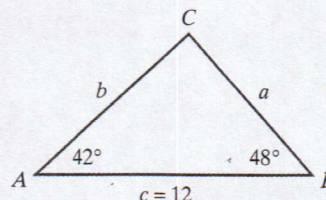
Exercise Set 7.1**Practice Exercises**

In Exercises 1–8, solve each triangle. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

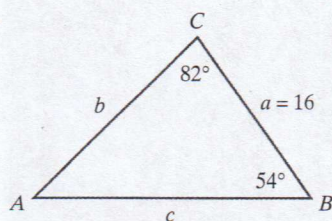
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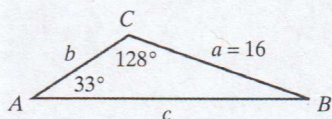
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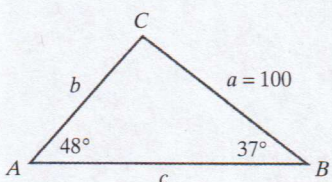
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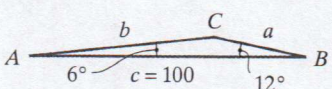
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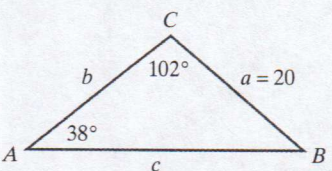
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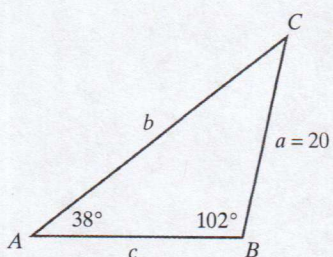
6.



7.



8.



In Exercises 9–16, solve each triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

9. $A = 44^\circ$, $B = 25^\circ$, $a = 12$
10. $A = 56^\circ$, $C = 24^\circ$, $a = 22$
11. $B = 85^\circ$, $C = 15^\circ$, $b = 40$
12. $A = 85^\circ$, $B = 35^\circ$, $c = 30$
13. $A = 115^\circ$, $C = 35^\circ$, $c = 200$
14. $B = 5^\circ$, $C = 125^\circ$, $b = 200$
15. $A = 65^\circ$, $B = 65^\circ$, $c = 6$
16. $B = 80^\circ$, $C = 10^\circ$, $a = 8$

In Exercises 17–32, two sides and an angle (SSA) of a triangle are given. Determine whether the given measurements produce one triangle, two triangles, or no triangle at all. Solve each triangle that results. Round to the nearest tenth and the nearest degree for sides and angles, respectively.

17. $a = 20$, $b = 15$, $A = 40^\circ$
18. $a = 30$, $b = 20$, $A = 50^\circ$
19. $a = 10$, $c = 8.9$, $A = 63^\circ$
20. $a = 57.5$, $c = 49.8$, $A = 136^\circ$
21. $a = 42.1$, $c = 37$, $A = 112^\circ$
22. $a = 6.1$, $b = 4$, $A = 162^\circ$
23. $a = 10$, $b = 40$, $A = 30^\circ$
24. $a = 10$, $b = 30$, $A = 150^\circ$
25. $a = 16$, $b = 18$, $A = 60^\circ$
26. $a = 30$, $b = 40$, $A = 20^\circ$
27. $a = 12$, $b = 16.1$, $A = 37^\circ$
28. $a = 7$, $b = 28$, $A = 12^\circ$
29. $a = 22$, $c = 24.1$, $A = 58^\circ$
30. $a = 95$, $c = 125$, $A = 49^\circ$
31. $a = 9.3$, $b = 41$, $A = 18^\circ$
32. $a = 1.4$, $b = 2.9$, $A = 142^\circ$

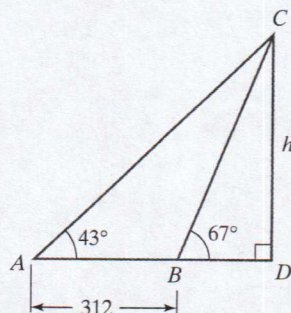
In Exercises 33–38, find the area of the triangle having the given measurements. Round to the nearest square unit.

33. $A = 48^\circ$, $b = 20$ feet, $c = 40$ feet
34. $A = 22^\circ$, $b = 20$ feet, $c = 50$ feet
35. $B = 36^\circ$, $a = 3$ yards, $c = 6$ yards
36. $B = 125^\circ$, $a = 8$ yards, $c = 5$ yards
37. $C = 124^\circ$, $a = 4$ meters, $b = 6$ meters
38. $C = 102^\circ$, $a = 16$ meters, $b = 20$ meters

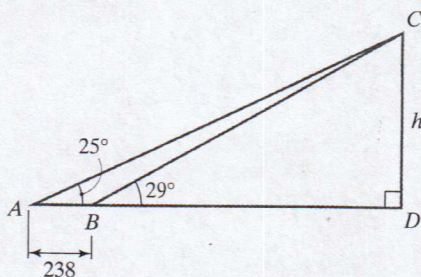
Practice Plus

In Exercises 39–40, find h to the nearest tenth.

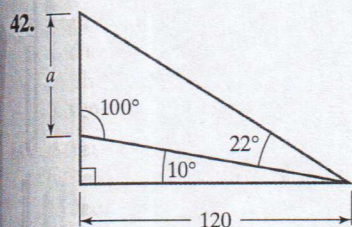
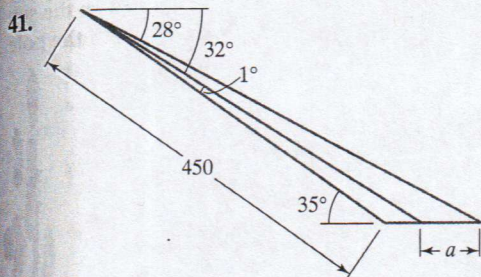
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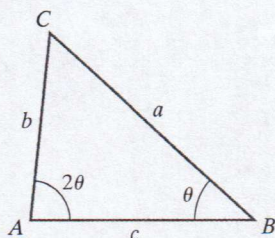
40.



In Exercises 41–42, find a to the nearest tenth.



In Exercises 43–44, use the given measurements to solve the following triangle. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.



43. $a = 300, b = 200$

44. $a = 400, b = 300$

In Exercises 45–46, find the area of the triangle with the given vertices. Round to the nearest square unit.

45. $(-3, -2), (2, -2), (1, 2)$

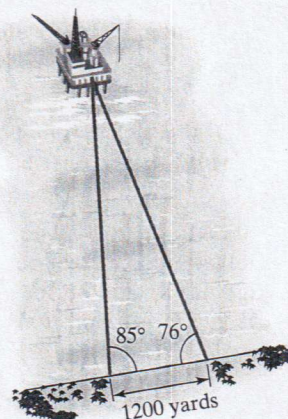
46. $(-2, -3), (-2, 2), (2, 1)$

Application Exercises

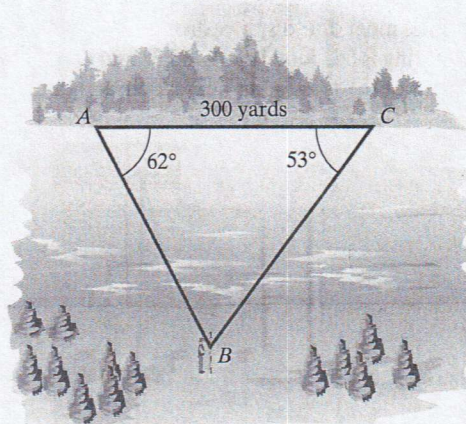
47. Two fire-lookout stations are 10 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is $N25^\circ E$ and the bearing of the fire from station B is $N56^\circ W$. How far, to the nearest tenth of a mile, is the fire from each lookout station?

48. The Federal Communications Commission is attempting to locate an illegal radio station. It sets up two monitoring stations, A and B, with station B 40 miles east of station A. Station A measures the illegal signal from the radio station as coming from a direction of 48° east of north. Station B measures the signal as coming from a point 34° west of north. How far is the illegal radio station from monitoring stations A and B? Round to the nearest tenth of a mile.

49. The figure shows a 1200-yard-long sand beach and an oil platform in the ocean. The angle made with the platform from one end of the beach is 85° and from the other end is 76° . Find the distance of the oil platform, to the nearest tenth of a yard, from each end of the beach.



50. A surveyor needs to determine the distance between two points that lie on opposite banks of a river. The figure shows that 300 yards are measured along one bank. The angles from each end of this line segment to a point on the opposite bank are 62° and 53° . Find the distance between A and B to the nearest tenth of a yard.



51. The Leaning Tower of Pisa in Italy leans at an angle of about 84.7° . The figure shows that 171 feet from the base of the tower, the angle of elevation to the top is 50° . Find the distance, to the nearest tenth of a foot, from the base to the top of the tower.

