We use a calculator in the degree mode to find θ.

Pressing [2nd] [TAN] accesses the inverse tangent key, [TAN⁻¹].

Many Scientific Calculators:

[( ] 21 [÷] 25 [ )] [2nd] [TAN]

Many Graphing Calculators:

[2nd] [TAN] [( ] 21 [÷] 25 [ )] [ENTER]

The display should show approximately 40. Thus, the angle of elevation of the sun is approximately 40°.

Check Point 10 A flagpole that is 14 meters tall casts a shadow 10 meters long. Find the angle of elevation of the sun to the nearest degree.

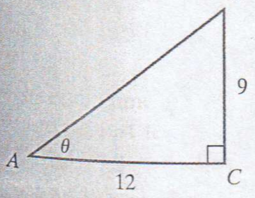
The Mountain Man

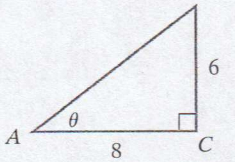
In the 1930s, a *National Geographic* team headed by Brad Washburn used trigonometry to create a map of the 5000-square-mile region of the Yukon, near the Canadian border. The team started with aerial photography. By drawing a network of angles on the photographs, the approximate locations of the major mountains and their rough heights were determined. The expedition then spent three months on foot to find the exact heights. Team members established two base points a known distance apart, one directly under the mountain's peak. By measuring the angle of elevation from one of the base points to the peak, the tangent function was used to determine the peak's height. The Yukon expedition was a major advance in the way maps are made.

Exercise Set 5.2

Practice Exercises

In Exercises 1–8, use the Pythagorean Theorem to find the length of the missing side of each right triangle. Then find the value of each of the six trigonometric functions of θ.

1.


2.


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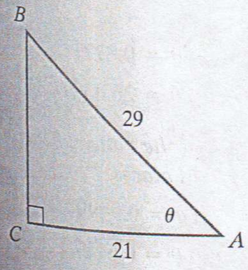

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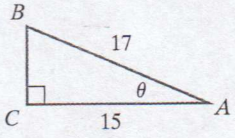

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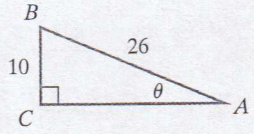

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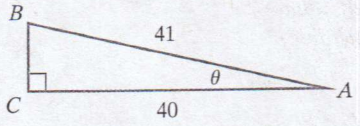

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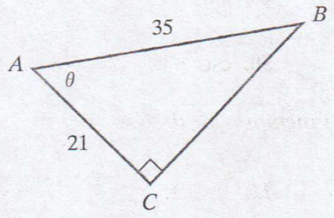

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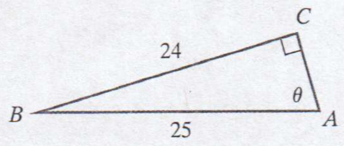
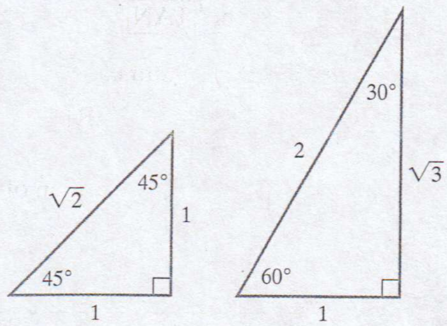


In Exercises 9–16, use the given triangles to evaluate each expression. If necessary, express the value without a square root in the denominator by rationalizing the denominator.



9. $\cos 30°$

10. $\tan 30°$

11. $\sec 45°$

12. $\csc 45°$

13. $\tan \frac{\pi}{3}$

14. $\cot \frac{\pi}{3}$

15. $\sin \frac{\pi}{4} - \cos \frac{\pi}{4}$

16. $\tan \frac{\pi}{4} + \csc \frac{\pi}{6}$

In Exercises 17–20, θ is an acute angle and $\sin \theta$ and $\cos \theta$ are given. Use identities to find $\tan \theta$, $\csc \theta$, $\sec \theta$, and $\cot \theta$. Where necessary, rationalize denominators.

17. $\sin \theta = \frac{8}{17}$, $\cos \theta = \frac{15}{17}$

18. $\sin \theta = \frac{3}{5}$, $\cos \theta = \frac{4}{5}$

19. $\sin \theta = \frac{1}{3}$, $\cos \theta = \frac{2\sqrt{2}}{3}$

20. $\sin \theta = \frac{6}{7}$, $\cos \theta = \frac{\sqrt{13}}{7}$

In Exercises 21–24, θ is an acute angle and $\sin \theta$ is given. Use the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ to find $\cos \theta$.

21. $\sin \theta = \frac{6}{7}$

22. $\sin \theta = \frac{7}{8}$

23. $\sin \theta = \frac{\sqrt{39}}{8}$

24. $\sin \theta = \frac{\sqrt{21}}{5}$

In Exercises 25–30, use an identity to find the value of each expression. Do not use a calculator.

25. $\sin 37° \csc 37°$

26. $\cos 53° \sec 53°$

27. $\sin^2 \frac{\pi}{9} + \cos^2 \frac{\pi}{9}$

28. $\sin^2 \frac{\pi}{10} + \cos^2 \frac{\pi}{10}$

29. $\sec^2 23° - \tan^2 23°$

30. $\csc^2 63° - \cot^2 63°$

In Exercises 31–38, find a cofunction with the same value as the given expression.

31. $\sin 7°$

32. $\sin 19°$

33. $\csc 25°$

34. $\csc 35°$

35. $\tan \frac{\pi}{9}$

36. $\tan \frac{\pi}{7}$

37. $\cos \frac{2\pi}{5}$

38. $\cos \frac{3\pi}{8}$

In Exercises 39–48, use a calculator to find the value of the trigonometric function to four decimal places.

39. $\sin 38°$

40. $\cos 21°$

41. $\tan 32.7°$

42. $\tan 52.6°$

43. $\csc 17°$

44. $\sec 55°$

45. $\cos \frac{\pi}{10}$

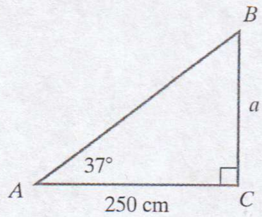
46. $\sin \frac{3\pi}{10}$

47. $\cot \frac{\pi}{12}$

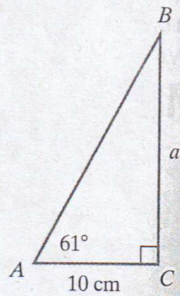
48. $\cot \frac{\pi}{18}$

In Exercises 49–54, find the measure of the side of the right triangle whose length is designated by a lowercase letter. Round answers to the nearest whole number.

49.



50.



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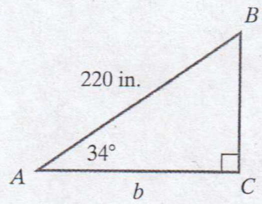
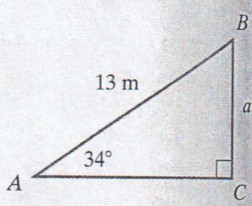


52.



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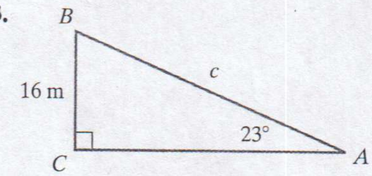
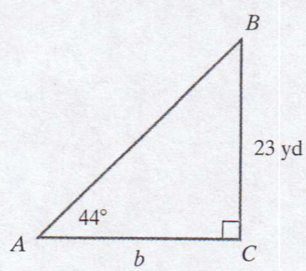


54.



In Exercises 55–58, use a calculator to find the value of the acute angle θ to the nearest degree.

55. $\sin \theta = 0.2974$

56. $\cos \theta = 0.8771$

57. $\tan \theta = 4.6252$

58. $\tan \theta = 26.0307$

In Exercises 59–62, use a calculator to find the value of the acute angle θ in radians, rounded to three decimal places.

59. $\cos \theta = 0.4112$

60. $\sin \theta = 0.9499$

61. $\tan \theta = 0.4169$

62. $\tan \theta = 0.5117$

Practice Plus

In Exercises 63–68, find the exact value of each expression. Do not use a calculator.

63. $\dfrac{\tan\frac{\pi}{3}}{2} - \dfrac{1}{\sec\frac{\pi}{6}}$

64. $\dfrac{1}{\cot\frac{\pi}{4}} - \dfrac{2}{\csc\frac{\pi}{6}}$

65. $1 + \sin^2 40° + \sin^2 50°$

66. $1 - \tan^2 10° + \csc^2 80°$

67. $\csc 37° \sec 53° - \tan 53° \cot 37°$

68. $\cos 12° \sin 78° + \cos 78° \sin 12°$

In Exercises 69–70, express the exact value of each function as a single fraction. Do not use a calculator.

69. If $f(\theta) = 2\cos\theta - \cos 2\theta$, find $f\left(\frac{\pi}{6}\right)$.

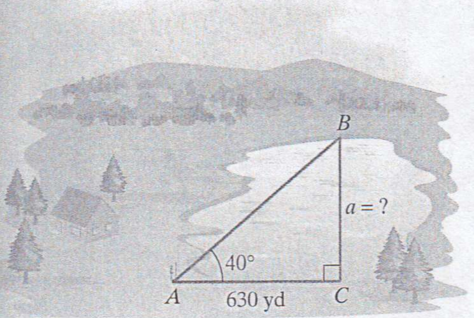
70. If $f(\theta) = 2\sin\theta - \sin\frac{\theta}{2}$, find $f\left(\frac{\pi}{3}\right)$.

71. If θ is an acute angle and $\cot\theta = \frac{1}{4}$, find $\tan\left(\frac{\pi}{2} - \theta\right)$.

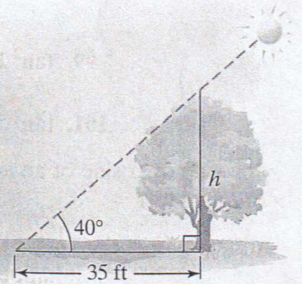
72. If θ is an acute angle and $\cos\theta = \frac{1}{3}$, find $\csc\left(\frac{\pi}{2} - \theta\right)$.

Application Exercises

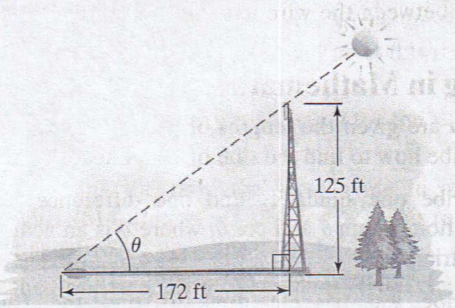
73. To find the distance across a lake, a surveyor took the measurements shown in the figure. Use these measurements to determine how far it is across the lake. Round to the nearest yard.



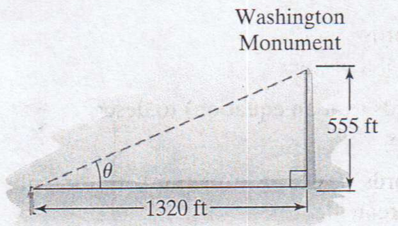
74. At a certain time of day, the angle of elevation of the sun is 40°. To the nearest foot, find the height of a tree whose shadow is 35 feet long.



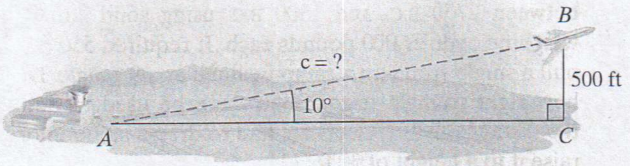
75. A tower that is 125 feet tall casts a shadow 172 feet long. Find the angle of elevation of the sun to the nearest degree.



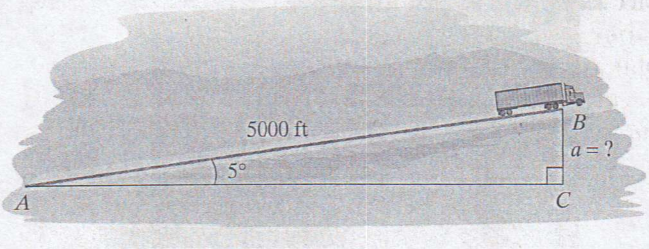
76. The Washington Monument is 555 feet high. If you are standing one quarter of a mile, or 1320 feet, from the base of the monument and looking to the top, find the angle of elevation to the nearest degree.



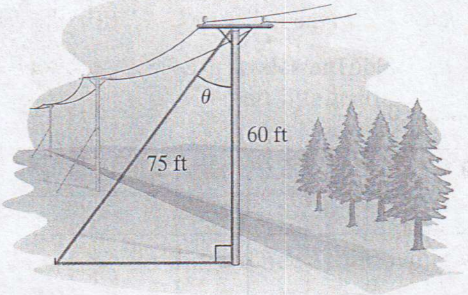
77. A plane rises from take-off and flies at an angle of 10° with the horizontal runway. When it has gained 500 feet, find the distance, to the nearest foot, the plane has flown.



78. A road is inclined at an angle of 5°. After driving 5000 feet along this road, find the driver's increase in altitude. Round to the nearest foot.



79. A telephone pole is 60 feet tall. A guy wire 75 feet long is attached from the ground to the top of the pole. Find the angle between the wire and the pole to the nearest degree.



80. A telephone pole is 55 feet tall. A guy wire 80 feet long is attached from the ground to the top of the pole. Find the angle between the wire and the pole to the nearest degree.

Writing in Mathematics

81. If you are given the lengths of the sides of a right triangle, describe how to find the sine of either acute angle.

82. Describe one similarity and one difference between the definitions of $\sin\theta$ and $\cos\theta$, where θ is an acute angle of a right triangle.

83. Describe the triangle used to find the trigonometric functions of 45°.

84. Describe the triangle used to find the trigonometric functions of 30° and 60°.

85. What is a trigonometric identity?

86. Use words (not an equation) to describe one of the reciprocal identities.

87. Use words (not an equation) to describe one of the quotient identities.

88. Use words (not an equation) to describe one of the Pythagorean identities.

89. Describe a relationship among trigonometric functions that is based on angles that are complements.

90. Describe what is meant by an angle of elevation and an angle of depression.

91. Stonehenge, the famous "stone circle" in England, was built between 2750 B.C. and 1300 B.C. using solid stone blocks weighing over 99,000 pounds each. It required 550 people to pull a single stone up a ramp inclined at a 9° angle. Describe how right triangle trigonometry can be used to determine the distance the 550 workers had to drag a stone in order to raise it to a height of 30 feet.

Technology Exercises

92. Use a calculator in the radian mode to fill in the values in the following table. Then draw a conclusion about $\frac{\sin\theta}{\theta}$ as θ approaches 0.

θ	0.4	0.3	0.2	0.1	0.01	0.001	0.0001	0.00001
$\sin\theta$								
$\frac{\sin\theta}{\theta}$								

93. Use a calculator in the radian mode to fill in the values in the following table. Then draw a conclusion about $\frac{\cos\theta - 1}{\theta}$ as θ approaches 0.

θ	0.4	0.3	0.2	0.1	0.01	0.001	0.0001	0.00001
$\cos\theta$								
$\frac{\cos\theta - 1}{\theta}$								

Critical Thinking Exercises

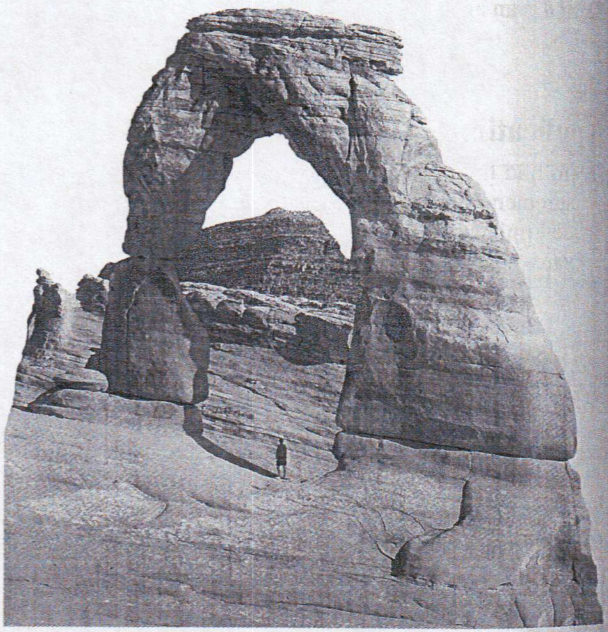
Make Sense? *In Exercises 94–97, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

94. For a given angle θ, I found a slight increase in $\sin\theta$ as the size of the triangle increased.

95. Although I can use an isosceles right triangle to determine the exact value of $\sin\frac{\pi}{4}$, I can also use my calculator to obtain this value.

96. I can rewrite $\tan\theta$ as $\frac{1}{\cot\theta}$, as well as $\frac{\sin\theta}{\cos\theta}$.

97. Standing under this arch, I can determine its height by measuring the angle of elevation to the top of the arch and my distance to a point directly under the arch.



Delicate Arch in Arches National Park, Utah

In Exercises 98–101, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

98. $\frac{\tan 45°}{\tan 15°} = \tan 3°$

99. $\tan^2 15° - \sec^2 15° = -1$

100. $\sin 45° + \cos 45° = 1$

101. $\tan^2 5° = \tan 25°$

102. Explain why the sine or cosine of an acute angle cannot be greater than or equal to 1.

103. Describe what happens to the tangent of an acute angle as the angle gets close to 90°. What happens at 90°?