Exercise Set 3.5

Practice Exercises

In Exercises 1–8, find the domain of each rational function.

1. $f(x) = \dfrac{5x}{x - 4}$

2. $f(x) = \dfrac{7x}{x - 8}$

3. $g(x) = \dfrac{3x^2}{(x - 5)(x + 4)}$

4. $g(x) = \dfrac{2x^2}{(x - 2)(x + 6)}$

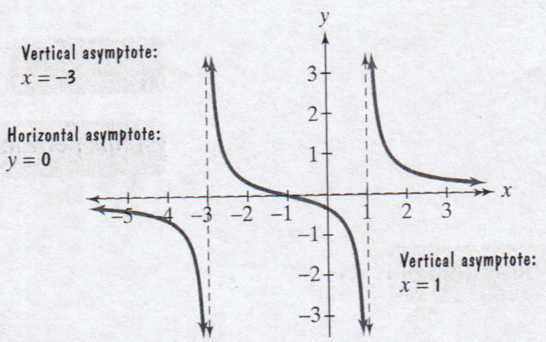
5. $h(x) = \dfrac{x + 7}{x^2 - 49}$

6. $h(x) = \dfrac{x + 8}{x^2 - 64}$

7. $f(x) = \dfrac{x + 7}{x^2 + 49}$

8. $f(x) = \dfrac{x + 8}{x^2 + 64}$

Use the graph of the rational function in the figure shown to complete each statement in Exercises 9–14.



9. As $x \to -3^-$, $f(x) \to$ ____.

10. As $x \to -3^+$, $f(x) \to$ ____.

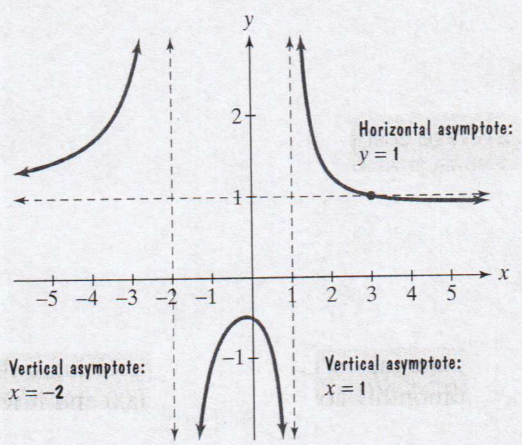
11. As $x \to 1^-$, $f(x) \to$ ____.

12. As $x \to 1^+$, $f(x) \to$ ____.

13. As $x \to -\infty$, $f(x) \to$ ____.

14. As $x \to \infty$, $f(x) \to$ ____.

Use the graph of the rational function in the figure shown to complete each statement in Exercises 15–20.



15. As $x \to 1^+$, $f(x) \to$ ____.

16. As $x \to 1^-$, $f(x) \to$ ____.

17. As $x \to -2^+$, $f(x) \to$ ____.

18. As $x \to -2^-$, $f(x) \to$ ____.

19. As $x \to \infty$, $f(x) \to$ ____.

20. As $x \to -\infty$, $f(x) \to$ ____.

In Exercises 21–28, find the vertical asymptotes, if any, of the graph of each rational function.

21. $f(x) = \dfrac{x}{x + 4}$

22. $f(x) = \dfrac{x}{x - 3}$

23. $g(x) = \dfrac{x + 3}{x(x + 4)}$

24. $g(x) = \dfrac{x + 3}{x(x - 3)}$

25. $h(x) = \dfrac{x}{x(x + 4)}$

26. $h(x) = \dfrac{x}{x(x - 3)}$

27. $r(x) = \dfrac{x}{x^2 + 4}$

28. $r(x) = \dfrac{x}{x^2 + 3}$

In Exercises 29–36, find the horizontal asymptote, if any, of the graph of each rational function.

29. $f(x) = \dfrac{12x}{3x^2 + 1}$

30. $f(x) = \dfrac{15x}{3x^2 + 1}$

31. $g(x) = \dfrac{12x^2}{3x^2 + 1}$

32. $g(x) = \dfrac{15x^2}{3x^2 + 1}$

33. $h(x) = \dfrac{12x^3}{3x^2 + 1}$

34. $h(x) = \dfrac{15x^3}{3x^2 + 1}$

35. $f(x) = \dfrac{-2x + 1}{3x + 5}$

36. $f(x) = \dfrac{-3x + 7}{5x - 2}$

In Exercises 37–48, use transformations of $f(x) = \frac{1}{x}$ or $f(x) = \frac{1}{x^2}$ to graph each rational function.

37. $g(x) = \dfrac{1}{x - 1}$

38. $g(x) = \dfrac{1}{x - 2}$

39. $h(x) = \dfrac{1}{x} + 2$

40. $h(x) = \dfrac{1}{x} + 1$

41. $g(x) = \dfrac{1}{x + 1} - 2$

42. $g(x) = \dfrac{1}{x + 2} - 2$

43. $g(x) = \dfrac{1}{(x + 2)^2}$

44. $g(x) = \dfrac{1}{(x + 1)^2}$

45. $h(x) = \dfrac{1}{x^2} - 4$

46. $h(x) = \dfrac{1}{x^2} - 3$

47. $h(x) = \dfrac{1}{(x - 3)^2} + 1$

48. $h(x) = \dfrac{1}{(x - 3)^2} + 2$

In Exercises 49–70, follow the seven steps on page 373 to graph each rational function.

49. $f(x) = \dfrac{4x}{x - 2}$

50. $f(x) = \dfrac{3x}{x - 1}$

51. $f(x) = \dfrac{2x}{x^2 - 4}$

52. $f(x) = \dfrac{4x}{x^2 - 1}$

53. $f(x) = \dfrac{2x^2}{x^2 - 1}$

54. $f(x) = \dfrac{4x^2}{x^2 - 9}$

55. $f(x) = \dfrac{-x}{x + 1}$

56. $f(x) = \dfrac{-3x}{x + 2}$

57. $f(x) = -\dfrac{1}{x^2 - 4}$

58. $f(x) = -\dfrac{2}{x^2 - 1}$

59. $f(x) = \dfrac{2}{x^2 + x - 2}$

60. $f(x) = \dfrac{-2}{x^2 - x - 2}$

61. $f(x) = \dfrac{2x^2}{x^2 + 4}$

62. $f(x) = \dfrac{4x^2}{x^2 + 1}$

63. $f(x) = \dfrac{x + 2}{x^2 + x - 6}$

64. $f(x) = \dfrac{x - 4}{x^2 - x - 6}$

65. $f(x) = \dfrac{x^4}{x^2 + 2}$

66. $f(x) = \dfrac{2x^4}{x^2 + 1}$

67. $f(x) = \dfrac{x^2 + x - 12}{x^2 - 4}$

68. $f(x) = \dfrac{x^2}{x^2 + x - 6}$

69. $f(x) = \dfrac{3x^2 + x - 4}{2x^2 - 5x}$

70. $f(x) = \dfrac{x^2 - 4x + 3}{(x + 1)^2}$

In Exercises 71–78, **a.** *Find the slant asymptote of the graph of each rational function and* **b.** *Follow the seven-step strategy and use the slant asymptote to graph each rational function.*

71. $f(x) = \dfrac{x^2 - 1}{x}$

72. $f(x) = \dfrac{x^2 - 4}{x}$

73. $f(x) = \dfrac{x^2 + 1}{x}$

74. $f(x) = \dfrac{x^2 + 4}{x}$

75. $f(x) = \dfrac{x^2 + x - 6}{x - 3}$

76. $f(x) = \dfrac{x^2 - x + 1}{x - 1}$

77. $f(x) = \dfrac{x^3 + 1}{x^2 + 2x}$

78. $f(x) = \dfrac{x^3 - 1}{x^2 - 9}$

Practice Plus

In Exercises 79–84, the equation for f is given by the simplified expression that results after performing the indicated operation. Write the equation for f and then graph the function.

79. $\dfrac{5x^2}{x^2 - 4} \cdot \dfrac{x^2 + 4x + 4}{10x^3}$

80. $\dfrac{x - 5}{10x - 2} \div \dfrac{x^2 - 10x + 25}{25x^2 - 1}$

81. $\dfrac{x}{2x + 6} - \dfrac{9}{x^2 - 9}$

82. $\dfrac{2}{x^2 + 3x + 2} - \dfrac{4}{x^2 + 4x + 3}$

83. $\dfrac{1 - \dfrac{3}{x + 2}}{1 + \dfrac{1}{x - 2}}$

84. $\dfrac{x - \dfrac{1}{x}}{x + \dfrac{1}{x}}$

In Exercises 85–88, use long division to rewrite the equation for g in the form

$$\text{quotient} + \frac{\text{remainder}}{\text{divisor}}.$$

Then use this form of the function's equation and transformations of $f(x) = \frac{1}{x}$ to graph g.

85. $g(x) = \dfrac{2x + 7}{x + 3}$

86. $g(x) = \dfrac{3x + 7}{x + 2}$

87. $g(x) = \dfrac{3x - 7}{x - 2}$

88. $g(x) = \dfrac{2x - 9}{x - 4}$

Application Exercises

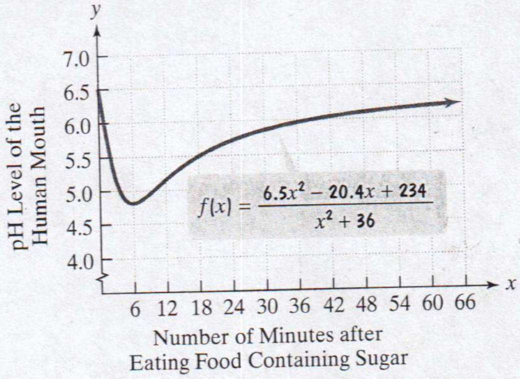
89. A company is planning to manufacture mountain bikes. The fixed monthly cost will be $100,000 and it will cost $100 to produce each bicycle.
   - **a.** Write the cost function, C, of producing x mountain bikes.
   - **b.** Write the average cost function, $\overline{C}$, of producing x mountain bikes.
   - **c.** Find and interpret $\overline{C}(500)$, $\overline{C}(1000)$, $\overline{C}(2000)$, and $\overline{C}(4000)$.
   - **d.** What is the horizontal asymptote for the graph of the average cost function, $\overline{C}$? Describe what this means in practical terms.

90. A company that manufactures running shoes has a fixed monthly cost of $300,000. It costs $30 to produce each pair of shoes.
   - **a.** Write the cost function, C, of producing x pairs of shoes.
   - **b.** Write the average cost function, $\overline{C}$, of producing x pairs of shoes.
   - **c.** Find and interpret $\overline{C}(1000)$, $\overline{C}(10{,}000)$, and $\overline{C}(100{,}000)$.
   - **d.** What is the horizontal asymptote for the graph of the average cost function, $\overline{C}$? Describe what this represents for the company.

91. The function

$$f(x) = \frac{6.5x^2 - 20.4x + 234}{x^2 + 36}$$

models the pH level, $f(x)$, of the human mouth x minutes after a person eats food containing sugar. The graph of this function is shown in the figure.

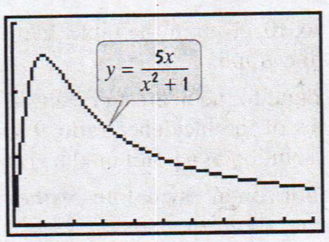


   - **a.** Use the graph to obtain a reasonable estimate, to the nearest tenth, of the pH level of the human mouth 42 minutes after a person eats food containing sugar.
   - **b.** After eating sugar, when is the pH level the lowest? Use the function's equation to determine the pH level, to the nearest tenth, at this time.
   - **c.** According to the graph, what is the normal pH level of the human mouth?
   - **d.** What is the equation of the horizontal asymptote associated with this function? Describe what this means in terms of the mouth's pH level over time.
   - **e.** Use the graph to describe what happens to the pH level during the first hour.

92. A drug is injected into a patient and the concentration of the drug in the bloodstream is monitored. The drug's concentration, $C(t)$, in milligrams per liter, after t hours is modeled by

$$C(t) = \frac{5t}{t^2 + 1}.$$

The graph of this rational function, obtained with a graphing utility, is shown in the figure.



[0, 10, 1] by [0, 3, 1]

a. Use the graph at the bottom of the previous page to obtain a reasonable estimate of the drug's concentration after 3 hours.

b. Use the function's equation displayed in the voice balloon by the graph to determine the drug's concentration after 3 hours.

c. Use the function's equation to find the horizontal asymptote for the graph. Describe what this means about the drug's concentration in the patient's bloodstream as time increases.

Among all deaths from a particular disease, the percentage that are smoking related (21–39 cigarettes per day) is a function of the disease's ***incidence ratio****. The incidence ratio describes the number of times more likely smokers are than nonsmokers to die from the disease. The following table shows the incidence ratios for heart disease and lung cancer for two age groups.*

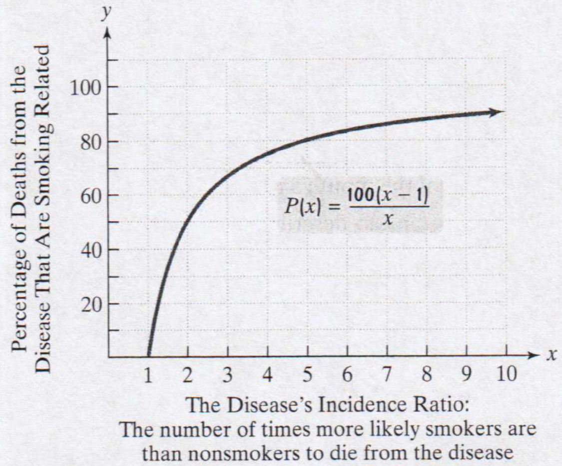
Incidence Ratios		
	Heart Disease	**Lung Cancer**
Ages 55–64	1.9	10
Ages 65–74	1.7	9

Source: Alexander M. Walker, *Observations and Inference,* Epidemiology Resources Inc., 1991.

For example, the incidence ratio of 9 in the table means that smokers between the ages of 65 and 74 are 9 times more likely than nonsmokers in the same age group to die from lung cancer. The rational function

$$P(x) = \frac{100(x - 1)}{x}$$

models the percentage of smoking-related deaths among all deaths from a disease, $P(x)$*, in terms of the disease's incidence ratio,* x*. The graph of the rational function is shown. Use this function to solve Exercises 93–96.*



93. Find $P(10)$. Describe what this means in terms of the incidence ratio, 10, given in the table. Identify your solution as a point on the graph.

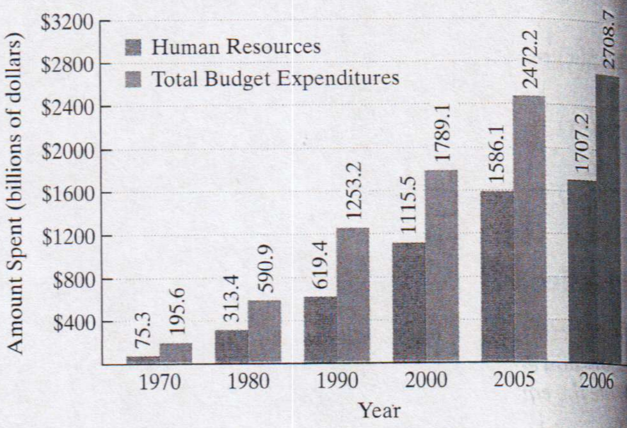
94. Find $P(9)$. Round to the nearest percent. Describe what this means in terms of the incidence ratio, 9, given in the table. Identify your solution as a point on the graph.

95. What is the horizontal asymptote of the graph? Describe what this means about the percentage of deaths caused by smoking with increasing incidence ratios.

96. According to the model and its graph, is there a disease for which all deaths are caused by smoking? Explain your answer.

97. The bar graph shows the amount, in billions of dollars, that the United States government spent on human resources and total budget outlays for six selected years. (Human resources include education, health, Medicare, Social Security, and veterans benefits and services.)

Federal Budget Expenditures on Human Resources



Source: Office of Management and Budget

The function $p(x) = 11x^2 + 40x + 1040$ models the amount, $p(x)$, in billions of dollars, that the United States government spent on human resources x years after 1970. The function $q(x) = 12x^2 + 230x + 2190$ models total budget expenditures, $q(x)$, in billions of dollars, x years after 1970.

a. Use p and q to write a rational function that models the fraction of total budget outlays spent on human resources x years after 1970.

b. Use the data displayed by the bar graph to find the percentage of federal expenditures spent on human resources in 2006. Round to the nearest percent.

c. Use the rational function from part (a) to find the percentage of federal expenditures spent on human resources in 2006. Round to the nearest percent. Does this underestimate or overestimate the actual percent that you found in part (b)? By how much?

d. What is the equation of the horizontal asymptote associated with the rational function in part (a)? If trends modeled by the function continue, what percentage of the federal budget will be spent on human resources over time? Round to the nearest percent. Does this projection seem realistic? Why or why not?

Writing in Mathematics

98. What is a rational function?

99. Use everyday language to describe the graph of a rational function f such that as $x \to -\infty$, $f(x) \to 3$.