Exercise Set 3.2

Practice Exercises

In Exercises 1–10, determine which functions are polynomial functions. For those that are, identify the degree.

1. $f(x) = 5x^2 + 6x^3$

2. $f(x) = 7x^2 + 9x^4$

3. $g(x) = 7x^5 - \pi x^3 + \frac{1}{5}x$

4. $g(x) = 6x^7 + \pi x^5 + \frac{2}{3}x$

5. $h(x) = 7x^3 + 2x^2 + \frac{1}{x}$

6. $h(x) = 8x^3 - x^2 + \frac{2}{x}$

7. $f(x) = x^{\frac{1}{2}} - 3x^2 + 5$

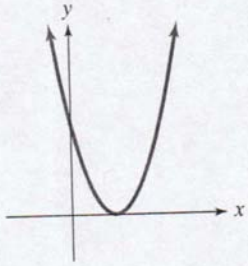
8. $f(x) = x^{\frac{1}{3}} - 4x^2 + 7$

9. $f(x) = \frac{x^2 + 7}{x^3}$

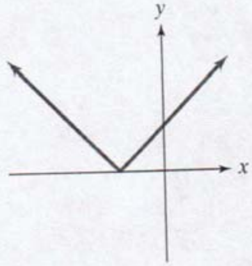
10. $f(x) = \frac{x^2 + 7}{3}$

In Exercises 11–14, identify which graphs are not those of polynomial functions.

11.



12.



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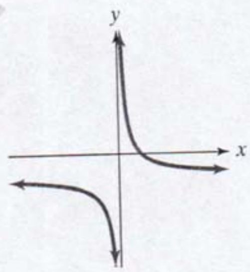

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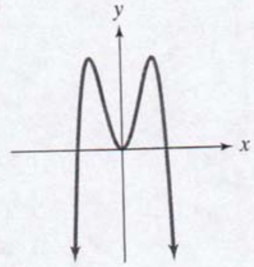


In Exercises 15–18, use the Leading Coefficient Test to determine the end behavior of the graph of the given polynomial function. Then use this end behavior to match the polynomial function with its graph. [The graphs are labeled (a) through (d).]

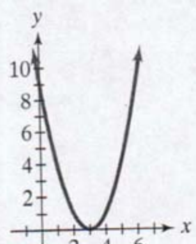
15. $f(x) = -x^4 + x^2$

16. $f(x) = x^3 - 4x^2$

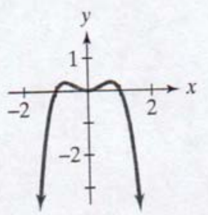
17. $f(x) = (x - 3)^2$

18. $f(x) = -x^3 - x^2 + 5x - 3$

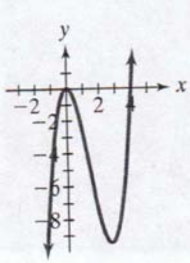
a.



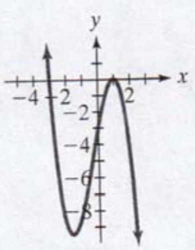
b.



c.



d.



In Exercises 19–24, use the Leading Coefficient Test to determine the end behavior of the graph of the polynomial function.

19. $f(x) = 5x^3 + 7x^2 - x + 9$

20. $f(x) = 11x^3 - 6x^2 + x + 3$

21. $f(x) = 5x^4 + 7x^2 - x + 9$

22. $f(x) = 11x^4 - 6x^2 + x + 3$

23. $f(x) = -5x^4 + 7x^2 - x + 9$

24. $f(x) = -11x^4 - 6x^2 + x + 3$

In Exercises 25–32, find the zeros for each polynomial function and give the multiplicity for each zero. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.

25. $f(x) = 2(x - 5)(x + 4)^2$

26. $f(x) = 3(x + 5)(x + 2)^2$

27. $f(x) = 4(x - 3)(x + 6)^3$

28. $f(x) = -3\left(x + \frac{1}{2}\right)(x - 4)^3$

29. $f(x) = x^3 - 2x^2 + x$

30. $f(x) = x^3 + 4x^2 + 4x$

31. $f(x) = x^3 + 7x^2 - 4x - 28$

32. $f(x) = x^3 + 5x^2 - 9x - 45$

In Exercises 33–40, use the Intermediate Value Theorem to show that each polynomial has a real zero between the given integers.

33. $f(x) = x^3 - x - 1$; between 1 and 2

34. $f(x) = x^3 - 4x^2 + 2$; between 0 and 1

35. $f(x) = 2x^4 - 4x^2 + 1$; between -1 and 0

36. $f(x) = x^4 + 6x^3 - 18x^2$; between 2 and 3

37. $f(x) = x^3 + x^2 - 2x + 1$; between -3 and -2

38. $f(x) = x^5 - x^3 - 1$; between 1 and 2

39. $f(x) = 3x^3 - 10x + 9$; between -3 and -2

40. $f(x) = 3x^3 - 8x^2 + x + 2$; between 2 and 3

In Exercises 41–64,

a. *Use the Leading Coefficient Test to determine the graph's end behavior.*

b. *Find the x-intercepts. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each intercept.*

c. *Find the y-intercept.*

d. *Determine whether the graph has y-axis symmetry, origin symmetry, or neither.*

e. *If necessary, find a few additional points and graph the function. Use the maximum number of turning points to check whether it is drawn correctly.*

41. $f(x) = x^3 + 2x^2 - x - 2$

42. $f(x) = x^3 + x^2 - 4x - 4$

43. $f(x) = x^4 - 9x^2$

44. $f(x) = x^4 - x^2$

45. $f(x) = -x^4 + 16x^2$

46. $f(x) = -x^4 + 4x^2$

47. $f(x) = x^4 - 2x^3 + x^2$

48. $f(x) = x^4 - 6x^3 + 9x^2$

49. $f(x) = -2x^4 + 4x^3$

50. $f(x) = -2x^4 + 2x^3$

51. $f(x) = 6x^3 - 9x - x^5$

52. $f(x) = 6x - x^3 - x^5$

53. $f(x) = 3x^2 - x^3$

54. $f(x) = \frac{1}{2} - \frac{1}{2}x^4$

55. $f(x) = -3(x - 1)^2(x^2 - 4)$

56. $f(x) = -2(x - 4)^2(x^2 - 25)$

57. $f(x) = x^2(x - 1)^3(x + 2)$

58. $f(x) = x^3(x + 2)^2(x + 1)$

59. $f(x) = -x^2(x - 1)(x + 3)$

60. $f(x) = -x^2(x + 2)(x - 2)$

61. $f(x) = -2x^3(x - 1)^2(x + 5)$

62. $f(x) = -3x^3(x - 1)^2(x + 3)$

63. $f(x) = (x - 2)^2(x + 4)(x - 1)$

64. $f(x) = (x + 3)(x + 1)^3(x + 4)$

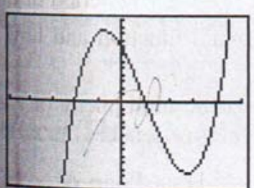
Practice Plus

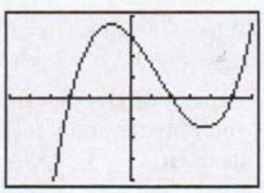
In Exercises 65–72, complete graphs of polynomial functions whose zeros are integers are shown.

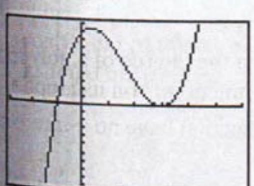
a. *Find the zeros and state whether the multiplicity of each zero is even or odd.*

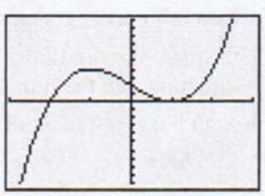
b. *Write an equation, expressed as the product of factors, of a polynomial function that might have each graph. Use a leading coefficient of 1 or −1, and make the degree of f as small as possible.*

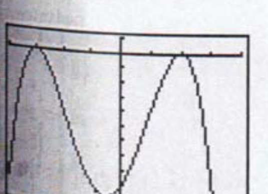
c. *Use both the equation in part (b) and the graph to find the y-intercept.*

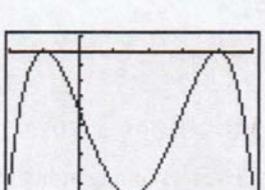
65.

[−5, 5, 1] by [−12, 12, 1]

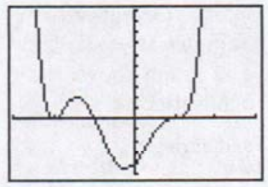
66.

[−6, 6, 1] by [−40, 40, 10]

67.

[−3, 6, 1] by [−10, 10, 1]

68.

[−3, 3, 1] by [−10, 10, 1]

69.

[−4, 4, 1] by [−40, 4, 4]

70.

[−2, 5, 1] by [−40, 4, 4]

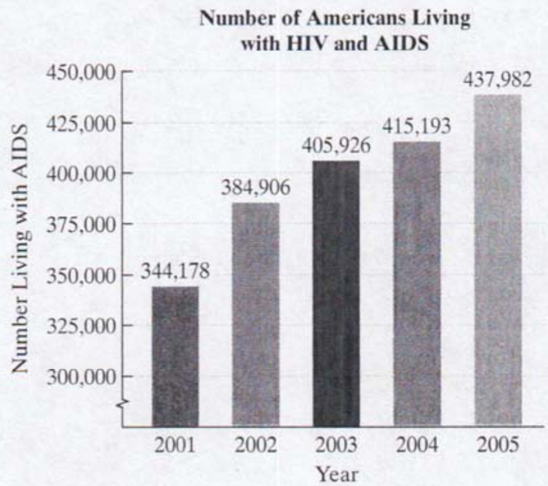
71.

[−3, 3, 1] by [−5, 10, 1]

72.

[−3, 3, 1] by [−5, 10, 1]

Application Exercises

The bar graph shows the number of Americans living with HIV and AIDS from 2001 through 2005.



Source: Department of Health and Human Services

The data in the bar graph can be modeled by the following second- and third-degree polynomial functions:

Number living with HIV and AIDS *x* years after 2000

$$f(x) = -3402x^2 + 42{,}203x + 308{,}453$$
$$g(x) = 2769x^3 - 28{,}324x^2 + 107{,}555x + 261{,}931.$$

Use these functions to solve Exercises 73–74.

73. **a.** Use both functions to find the number of Americans living with HIV and AIDS in 2003. Which function provides a better description for the actual number shown in the bar graph?

b. Consider the function from part (a) that serves as a better model for 2003. Use the Leading Coefficient Test to determine the end behavior to the right for the graph of this function. Will the function be useful in modeling the number of Americans living with HIV and AIDS over an extended period of time? Explain your answer.

74. **a.** Use both functions to find the number of Americans living with HIV and AIDS in 2005. Which function provides a better description for the actual number shown in the bar graph?

b. Consider the function from part (a) that serves as a better model for 2005. Use the Leading Coefficient Test to determine the end behavior to the right for the graph of this function. Based on this end behavior, can the function be used to model the number of Americans living with HIV and AIDS over an extended period of time? Explain your answer.