Check Point 6 Write in standard form and graph:

$$x^2 + y^2 + 4x - 4y - 1 = 0.$$

Exercise Set 2.8

Practice Exercises

In Exercises 1–18, find the distance between each pair of points. If necessary, round answers to two decimals places.

1. $(2,3)$ and $(14,8)$
2. $(5,1)$ and $(8,5)$
3. $(4,-1)$ and $(-6,3)$
4. $(2,-3)$ and $(-1,5)$
5. $(0,0)$ and $(-3,4)$
6. $(0,0)$ and $(3,-4)$
7. $(-2,-6)$ and $(3,-4)$
8. $(-4,-1)$ and $(2,-3)$
9. $(0,-3)$ and $(4,1)$
10. $(0,-2)$ and $(4,3)$
11. $(3.5,8.2)$ and $(-0.5,6.2)$
12. $(2.6,1.3)$ and $(1.6,-5.7)$
13. $\left(0,-\sqrt{3}\right)$ and $\left(\sqrt{5},0\right)$
14. $\left(0,-\sqrt{2}\right)$ and $\left(\sqrt{7},0\right)$
15. $\left(3\sqrt{3},\sqrt{5}\right)$ and $\left(-\sqrt{3},4\sqrt{5}\right)$
16. $\left(2\sqrt{3},\sqrt{6}\right)$ and $\left(-\sqrt{3},5\sqrt{6}\right)$
17. $\left(\frac{7}{3},\frac{1}{5}\right)$ and $\left(\frac{1}{3},\frac{6}{5}\right)$
18. $\left(-\frac{1}{4},-\frac{1}{7}\right)$ and $\left(\frac{3}{4},\frac{6}{7}\right)$

In Exercises 19–30, find the midpoint of each line segment with the given endpoints.

19. $(6,8)$ and $(2,4)$
20. $(10,4)$ and $(2,6)$
21. $(-2,-8)$ and $(-6,-2)$
22. $(-4,-7)$ and $(-1,-3)$
23. $(-3,-4)$ and $(6,-8)$
24. $(-2,-1)$ and $(-8,6)$
25. $\left(-\frac{7}{2},\frac{3}{2}\right)$ and $\left(-\frac{5}{2},-\frac{11}{2}\right)$
26. $\left(-\frac{2}{5},\frac{7}{15}\right)$ and $\left(-\frac{2}{5},-\frac{4}{15}\right)$
27. $\left(8,3\sqrt{5}\right)$ and $\left(-6,7\sqrt{5}\right)$
28. $\left(7\sqrt{3},-6\right)$ and $\left(3\sqrt{3},-2\right)$
29. $\left(\sqrt{18},-4\right)$ and $\left(\sqrt{2},4\right)$
30. $\left(\sqrt{50},-6\right)$ and $\left(\sqrt{2},6\right)$

In Exercises 31–40, write the standard form of the equation of the circle with the given center and radius.

31. Center $(0,0)$, $r = 7$
32. Center $(0,0)$, $r = 8$
33. Center $(3,2)$, $r = 5$
34. Center $(2,-1)$, $r = 4$
35. Center $(-1,4)$, $r = 2$
36. Center $(-3,5)$, $r = 3$
37. Center $(-3,-1)$, $r = \sqrt{3}$
38. Center $(-5,-3)$, $r = \sqrt{5}$
39. Center $(-4,0)$, $r = 10$
40. Center $(-2,0)$, $r = 6$

In Exercises 41–52, give the center and radius of the circle described by the equation and graph each equation. Use the graph to identify the relation's domain and range.

41. $x^2 + y^2 = 16$
42. $x^2 + y^2 = 49$
43. $(x-3)^2 + (y-1)^2 = 36$
44. $(x-2)^2 + (y-3)^2 = 16$
45. $(x+3)^2 + (y-2)^2 = 4$
46. $(x+1)^2 + (y-4)^2 = 25$
47. $(x+2)^2 + (y+2)^2 = 4$
48. $(x+4)^2 + (y+5)^2 = 36$
49. $x^2 + (y-1)^2 = 1$
50. $x^2 + (y-2)^2 = 4$
51. $(x+1)^2 + y^2 = 25$
52. $(x+2)^2 + y^2 = 16$

In Exercises 53–64, complete the square and write the equation in standard form. Then give the center and radius of each circle and graph the equation.

53. $x^2 + y^2 + 6x + 2y + 6 = 0$
54. $x^2 + y^2 + 8x + 4y + 16 = 0$
55. $x^2 + y^2 - 10x - 6y - 30 = 0$
56. $x^2 + y^2 - 4x - 12y - 9 = 0$
57. $x^2 + y^2 + 8x - 2y - 8 = 0$
58. $x^2 + y^2 + 12x - 6y - 4 = 0$
59. $x^2 - 2x + y^2 - 15 = 0$
60. $x^2 + y^2 - 6y - 7 = 0$
61. $x^2 + y^2 - x + 2y + 1 = 0$
62. $x^2 + y^2 + x + y - \frac{1}{2} = 0$
63. $x^2 + y^2 + 3x - 2y - 1 = 0$
64. $x^2 + y^2 + 3x + 5y + \frac{9}{4} = 0$

Practice Plus

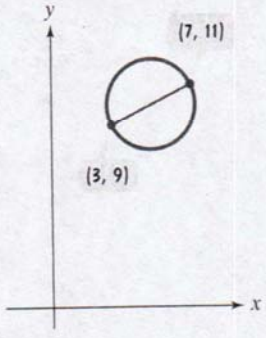
In Exercises 65–66, a line segment through the center of each circle intersects the circle at the points shown.

a. *Find the coordinates of the circle's center.*

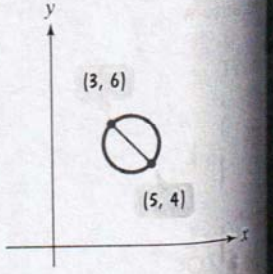
b. *Find the radius of the circle.*

c. *Use your answers from parts (a) and (b) to write the standard form of the circle's equation.*

65.



66.



In Exercises 67–70, graph both equations in the same rectangular coordinate system and find all points of intersection. Then show that these ordered pairs satisfy the equations.

67. $x^2 + y^2 = 16$
$x - y = 4$

68. $x^2 + y^2 = 9$
$x - y = 3$

69. $(x - 2)^2 + (y + 3)^2 = 4$
$y = x - 3$

70. $(x - 3)^2 + (y + 1)^2 = 9$
$y = x - 1$

Application Exercises

The cell phone screen shows coordinates of six cities from a rectangular coordinate system placed on North America by long-distance telephone companies. Each unit in this system represents $\sqrt{0.1}$ mile.

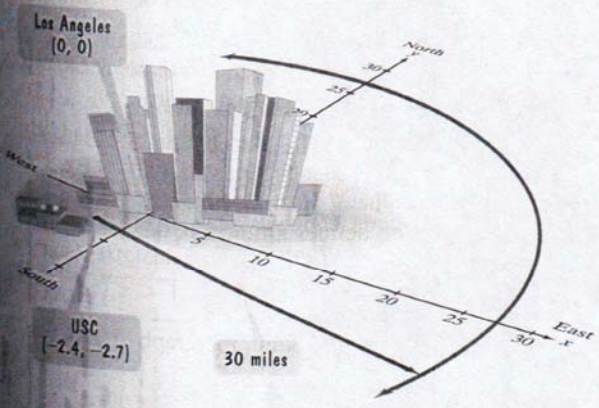
Source: Peter H. Dana

In Exercises 71–72, use this information to find the distance, to the nearest mile, between each pair of cities.

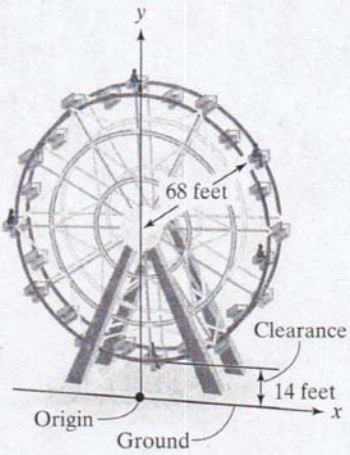
71. Boston and San Francisco

72. New Orleans and Houston

73. A rectangular coordinate system with coordinates in miles is placed with the origin at the center of Los Angeles. The figure indicates that the University of Southern California is located 2.4 miles west and 2.7 miles south of central Los Angeles. A seismograph on the campus shows that a small earthquake occurred. The quake's epicenter is estimated to be approximately 30 miles from the university. Write the standard form of the equation for the set of points that could be the epicenter of the quake.



74. The Ferris wheel in the figure has a radius of 68 feet. The clearance between the wheel and the ground is 14 feet. The rectangular coordinate system shown has its origin on the ground directly below the center of the wheel. Use the coordinate system to write the equation of the circular wheel.



Writing in Mathematics

75. In your own words, describe how to find the distance between two points in the rectangular coordinate system.

76. In your own words, describe how to find the midpoint of a line segment if its endpoints are known.

77. What is a circle? Without using variables, describe how the definition of a circle can be used to obtain a form of its equation.

78. Give an example of a circle's equation in standard form. Describe how to find the center and radius for this circle.

79. How is the standard form of a circle's equation obtained from its general form?

80. Does $(x - 3)^2 + (y - 5)^2 = 0$ represent the equation of a circle? If not, describe the graph of this equation.

81. Does $(x - 3)^2 + (y - 5)^2 = -25$ represent the equation of a circle? What sort of set is the graph of this equation?

82. Write and solve a problem about the flying time between a pair of cities shown on the cell phone screen for Exercises 71–72. Do not use the pairs in Exercise 71 or Exercise 72. Begin by determining a reasonable average speed, in miles per hour, for a jet flying between the cities.

Technology Exercises

In Exercises 83–85, use a graphing utility to graph each circle whose equation is given.

83. $x^2 + y^2 = 25$

84. $(y + 1)^2 = 36 - (x - 3)^2$

85. $x^2 + 10x + y^2 - 4y - 20 = 0$