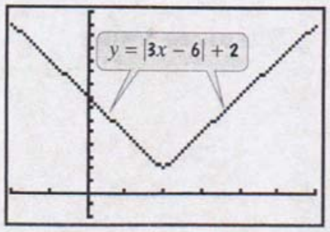


[−2, 6, 1] by [−2, 15, 1]

Figure 1.28 An absolute value equation whose graph has no x-intercepts

The absolute value of a number is never negative. Thus, if u is an algebraic expression and c is a negative number, then $|u| = c$ has no solution. For example, the equation $|3x - 6| = -2$ has no solution because $|3x - 6|$ cannot be negative. The solution set is $\varnothing$, the empty set. The graph of $y = |3x - 6| + 2$, shown in **Figure 1.28**, lies above the x-axis and has no x-intercepts.

The absolute value of 0 is 0. Thus, if u is an algebraic expression and $|u| = 0$, the solution is found by solving $u = 0$. For example, the solution of $|x - 2| = 0$ is obtained by solving $x - 2 = 0$. The solution is 2 and the solution set is $\{2\}$.

Applications

6 Solve problems modeled by equations.

EXAMPLE 10 Commercial Clutter

By 2005, the amount of "clutter," including commercials and plugs for other shows, had increased to the point where an "hour-long" drama on cable TV was 45.4 minutes. The graph in **Figure 1.29** shows the average number of nonprogram minutes in an hour of prime-time cable television. Although the minutes of clutter grew from 1996 through 2005, the growth was leveling off. The data can be modeled by the formula

$$M = 0.7\sqrt{x} + 12.5,$$

where M is the average number of nonprogram minutes in an hour of prime-time cable x years after 1996. Assuming the trend from 1996 through 2005 continues, use the model to project when there will be 15.1 cluttered minutes in every prime-time cable TV hour.

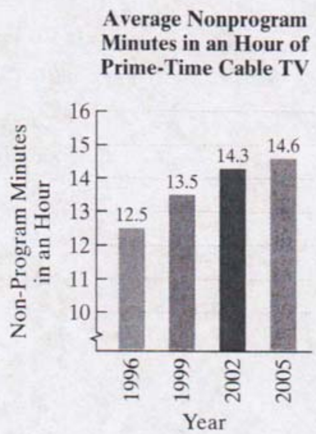


Figure 1.29
Source: Nielsen Monitor-Plus

Solution To find when there will be 15.1 cluttered minutes in every prime-time cable TV hour, substitute 15.1 for M in the given model. Then solve for x, the number of years after 1996.

$$M = 0.7\sqrt{x} + 12.5 \quad \text{This is the given formula.}$$

$$15.1 = 0.7\sqrt{x} + 12.5 \quad \text{Substitute 15.1 for } M.$$

$$2.6 = 0.7\sqrt{x} \quad \text{Subtract 12.5 from both sides.}$$

$$\frac{2.6}{0.7} = \sqrt{x} \quad \text{Divide both sides by 0.7.}$$

$$\left(\frac{2.6}{0.7}\right)^2 = (\sqrt{x})^2 \quad \text{Square both sides.}$$

$$14 \approx x \quad \text{Use a calculator.}$$

The model indicates that there will be 15.1 cluttered minutes in an hour approximately 14 years after 1996. Because $1996 + 14 = 2010$, this is projected to occur in 2010.

Check Point 10 Use the model in Example 10 to project when there will be 16 cluttered minutes in a prime-time cable TV hour.

Exercise Set 1.6

Practice Exercises

Solve each polynomial equation in Exercises 1–10 by factoring and then using the zero-product principle.

1. $3x^4 - 48x^2 = 0$ **2.** $5x^4 - 20x^2 = 0$

3. $3x^3 + 2x^2 = 12x + 8$ **4.** $4x^3 - 12x^2 = 9x - 27$

5. $2x - 3 = 8x^3 - 12x^2$ **6.** $x + 1 = 9x^3 + 9x^2$

7. $4y^3 - 2 = y - 8y^2$ **8.** $9y^3 + 8 = 4y + 18y^2$

9. $2x^4 = 16x$ **10.** $3x^4 = 81x$

Solve each radical equation in Exercises 11–30. Check all proposed solutions.

11. $\sqrt{3x + 18} = x$ **12.** $\sqrt{20 - 8x} = x$

13. $\sqrt{x + 3} = x - 3$ **14.** $\sqrt{x + 10} = x - 2$

15. $\sqrt{2x + 13} = x + 7$
16. $\sqrt{6x + 1} = x - 1$
17. $x - \sqrt{2x + 5} = 5$
18. $x - \sqrt{x + 11} = 1$
19. $\sqrt{2x + 19} - 8 = x$
20. $\sqrt{2x + 15} - 6 = x$
21. $\sqrt{3x} + 10 = x + 4$
22. $\sqrt{x} - 3 = x - 9$
23. $\sqrt{x + 8} - \sqrt{x - 4} = 2$
24. $\sqrt{x + 5} - \sqrt{x - 3} = 2$
25. $\sqrt{x - 5} - \sqrt{x - 8} = 3$
26. $\sqrt{2x - 3} - \sqrt{x - 2} = 1$
27. $\sqrt{2x + 3} + \sqrt{x - 2} = 2$
28. $\sqrt{x + 2} + \sqrt{3x + 7} = 1$
29. $\sqrt{3\sqrt{x + 1}} = \sqrt{3x - 5}$
30. $\sqrt{1 + 4\sqrt{x}} = 1 + \sqrt{x}$

Solve each equation with rational exponents in Exercises 31–40. Check all proposed solutions.

31. $x^{\frac{3}{2}} = 8$
32. $x^{\frac{3}{2}} = 27$
33. $(x - 4)^{\frac{3}{2}} = 27$
34. $(x + 5)^{\frac{3}{2}} = 8$
35. $6x^{\frac{5}{2}} - 12 = 0$
36. $8x^{\frac{5}{3}} - 24 = 0$
37. $(x - 4)^{\frac{2}{3}} = 16$
38. $(x + 5)^{\frac{2}{3}} = 4$
39. $(x^2 - x - 4)^{\frac{3}{4}} - 2 = 6$
40. $(x^2 - 3x + 3)^{\frac{3}{2}} - 1 = 0$

Solve each equation in Exercises 41–60 by making an appropriate substitution.

41. $x^4 - 5x^2 + 4 = 0$
42. $x^4 - 13x^2 + 36 = 0$
43. $9x^4 = 25x^2 - 16$
44. $4x^4 = 13x^2 - 9$
45. $x - 13\sqrt{x} + 40 = 0$
46. $2x - 7\sqrt{x} - 30 = 0$
47. $x^{-2} - x^{-1} - 20 = 0$
48. $x^{-2} - x^{-1} - 6 = 0$
49. $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$
50. $2x^{\frac{2}{3}} + 7x^{\frac{1}{3}} - 15 = 0$
51. $x^{\frac{3}{2}} - 2x^{\frac{3}{4}} + 1 = 0$
52. $x^{\frac{2}{5}} + x^{\frac{1}{5}} - 6 = 0$
53. $2x - 3x^{\frac{1}{2}} + 1 = 0$
54. $x + 3x^{\frac{1}{2}} - 4 = 0$
55. $(x - 5)^2 - 4(x - 5) - 21 = 0$
56. $(x + 3)^2 + 7(x + 3) - 18 = 0$
57. $(x^2 - x)^2 - 14(x^2 - x) + 24 = 0$
58. $(x^2 - 2x)^2 - 11(x^2 - 2x) + 24 = 0$
59. $\left(y - \frac{8}{y}\right)^2 + 5\left(y - \frac{8}{y}\right) - 14 = 0$
60. $\left(y - \frac{10}{y}\right)^2 + 6\left(y - \frac{10}{y}\right) - 27 = 0$

In Exercises 61–78, solve each absolute value equation or indicate that the equation has no solution.

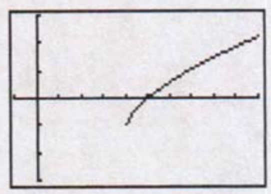
61. $|x| = 8$
62. $|x| = 6$
63. $|x - 2| = 7$
64. $|x + 1| = 5$
65. $|2x - 1| = 5$
66. $|2x - 3| = 11$
67. $2|3x - 2| = 14$
68. $3|2x - 1| = 21$
69. $7|5x| + 2 = 16$
70. $7|3x| + 2 = 16$
71. $2\left|4 - \frac{5}{2}x\right| + 6 = 18$
72. $4\left|1 - \frac{3}{4}x\right| + 7 = 10$
73. $|x + 1| + 5 = 3$
74. $|x + 1| + 6 = 2$
75. $|2x - 1| + 3 = 3$
76. $|3x - 2| + 4 = 4$

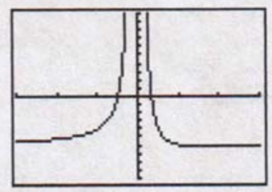
Hint for Exercises 77–78: Absolute value expressions are equal when the expressions inside the absolute value bars are equal to or opposites of each other.

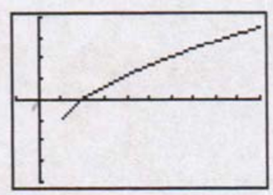
77. $|3x - 1| = |x + 5|$
78. $|2x - 7| = |x + 3|$

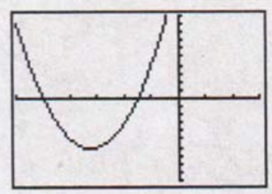
In Exercises 79–84, find the x-intercepts of the graph of each equation. Then use the x-intercepts to match the equation with its graph. [The graphs are labeled (a) through (f).]

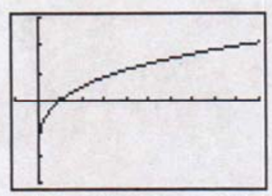
79. $y = \sqrt{x + 2} + \sqrt{x - 1} - 3$
80. $y = \sqrt{x - 4} + \sqrt{x + 4} - 4$
81. $y = x^{\frac{1}{3}} + 2x^{\frac{1}{6}} - 3$
82. $y = x^{-2} - x^{-1} - 6$
83. $y = (x + 2)^2 - 9(x + 2) + 20$
84. $y = 2(x + 2)^2 + 5(x + 2) - 3$

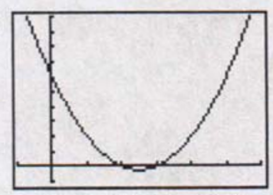
a.

[−1, 10, 1] by [−3, 3, 1]

b.

[−3, 3, 1] by [−10, 10, 1]

c.

[−1, 10, 1] by [−4, 4, 1]

d.

[−6, 3, 1] by [−10, 10, 1]

e.

[−1, 10, 1] by [−3, 3, 1]

f.

[−1, 6, 1] by [−1, 10, 1]

In Exercises 85–94, find all values of x satisfying the given conditions.

85. $y = |5 - 4x|$ and $y = 11$.
86. $y = |2 - 3x|$ and $y = 13$.
87. $y = x + \sqrt{x + 5}$ and $y = 7$.
88. $y = x - \sqrt{x - 2}$ and $y = 4$.
89. $y = 2x^3 + x^2 - 8x + 2$ and $y = 6$.
90. $y = x^3 + 4x^2 - x + 6$ and $y = 10$.
91. $y = (x + 4)^{\frac{3}{2}}$ and $y = 8$.