Exercise Set 1.4

Practice Exercises

In Exercises 1–8, add or subtract as indicated and write the result in standard form.

1. $(7 + 2i) + (1 - 4i)$
2. $(-2 + 6i) + (4 - i)$
3. $(3 + 2i) - (5 - 7i)$
4. $(-7 + 5i) - (-9 - 11i)$
5. $6 - (-5 + 4i) - (-13 - i)$
6. $7 - (-9 + 2i) - (-17 - i)$
7. $8i - (14 - 9i)$
8. $15i - (12 - 11i)$

In Exercises 9–20, find each product and write the result in standard form.

9. $-3i(7i - 5)$
10. $-8i(2i - 7)$
11. $(-5 + 4i)(3 + i)$
12. $(-4 - 8i)(3 + i)$
13. $(7 - 5i)(-2 - 3i)$
14. $(8 - 4i)(-3 + 9i)$
15. $(3 + 5i)(3 - 5i)$
16. $(2 + 7i)(2 - 7i)$
17. $(-5 + i)(-5 - i)$
18. $(-7 - i)(-7 + i)$
19. $(2 + 3i)^2$
20. $(5 - 2i)^2$

In Exercises 21–28, divide and express the result in standard form.

21. $\dfrac{2}{3 - i}$
22. $\dfrac{3}{4 + i}$
23. $\dfrac{2i}{1 + i}$
24. $\dfrac{5i}{2 - i}$
25. $\dfrac{8i}{4 - 3i}$
26. $\dfrac{-6i}{3 + 2i}$
27. $\dfrac{2 + 3i}{2 + i}$
28. $\dfrac{3 - 4i}{4 + 3i}$

In Exercises 29–44, perform the indicated operations and write the result in standard form.

29. $\sqrt{-64} - \sqrt{-25}$
30. $\sqrt{-81} - \sqrt{-144}$
31. $5\sqrt{-16} + 3\sqrt{-81}$
32. $5\sqrt{-8} + 3\sqrt{-18}$
33. $\left(-2 + \sqrt{-4}\right)^2$
34. $\left(-5 - \sqrt{-9}\right)^2$
35. $\left(-3 - \sqrt{-7}\right)^2$
36. $\left(-2 + \sqrt{-11}\right)^2$
37. $\dfrac{-8 + \sqrt{-32}}{24}$
38. $\dfrac{-12 + \sqrt{-28}}{32}$
39. $\dfrac{-6 - \sqrt{-12}}{48}$
40. $\dfrac{-15 - \sqrt{-18}}{33}$
41. $\sqrt{-8}\left(\sqrt{-3} - \sqrt{5}\right)$
42. $\sqrt{-12}\left(\sqrt{-4} - \sqrt{2}\right)$
43. $\left(3\sqrt{-5}\right)\left(-4\sqrt{-12}\right)$
44. $\left(3\sqrt{-7}\right)\left(2\sqrt{-8}\right)$

Practice Plus

In Exercises 45–50, perform the indicated operation(s) and write the result in standard form.

45. $(2 - 3i)(1 - i) - (3 - i)(3 + i)$
46. $(8 + 9i)(2 - i) - (1 - i)(1 + i)$
47. $(2 + i)^2 - (3 - i)^2$
48. $(4 - i)^2 - (1 + 2i)^2$
49. $5\sqrt{-16} + 3\sqrt{-81}$
50. $5\sqrt{-8} + 3\sqrt{-18}$
51. Evaluate $x^2 - 2x + 2$ for $x = 1 + i$.
52. Evaluate $x^2 - 2x + 5$ for $x = 1 - 2i$.
53. Evaluate $\dfrac{x^2 + 19}{2 - x}$ for $x = 3i$.
54. Evaluate $\dfrac{x^2 + 11}{3 - x}$ for $x = 4i$.

Application Exercises

Complex numbers are used in electronics to describe the current in an electric circuit. Ohm's law relates the current in a circuit, I, in amperes, the voltage of the circuit, E, in volts, and the resistance of the circuit, R, in ohms, by the formula $E = IR$. Use this formula to solve Exercises 55–56.

55. Find E, the voltage of a circuit, if $I = (4 - 5i)$ amperes and $R = (3 + 7i)$ ohms.
56. Find E, the voltage of a circuit, if $I = (2 - 3i)$ amperes and $R = (3 + 5i)$ ohms.
57. The mathematician Girolamo Cardano is credited with the first use (in 1545) of negative square roots in solving the now-famous problem, "Find two numbers whose sum is 10 and whose product is 40." Show that the complex numbers $5 + i\sqrt{15}$ and $5 - i\sqrt{15}$ satisfy the conditions of the problem. (Cardano did not use the symbolism $i\sqrt{15}$ or even $\sqrt{-15}$. He wrote R.m 15 for $\sqrt{-15}$, meaning "radix minus 15." He regarded the numbers 5 + R.m 15 and 5 − R.m 15 as "fictitious" or "ghost numbers," and considered the problem "manifestly impossible." But in a mathematically adventurous spirit, he exclaimed, "Nevertheless, we will operate.")

Writing in Mathematics

58. What is i?
59. Explain how to add complex numbers. Provide an example with your explanation.
60. Explain how to multiply complex numbers and give an example.
61. What is the complex conjugate of $2 + 3i$? What happens when you multiply this complex number by its complex conjugate?
62. Explain how to divide complex numbers. Provide an example with your explanation.
63. Explain each of the three jokes in the cartoon on page 130.
64. A stand-up comedian uses algebra in some jokes, including one about a telephone recording that announces "You have just reached an imaginary number. Please multiply by i and dial again." Explain the joke.

Explain the error in Exercises 65–66.

65. $\sqrt{-9} + \sqrt{-16} = \sqrt{-25} = i\sqrt{25} = 5i$
66. $\left(\sqrt{-9}\right)^2 = \sqrt{-9} \cdot \sqrt{-9} = \sqrt{81} = 9$