$$\mathbf{c.} \ (a+b)(a+b)^k = (a+b) \begin{bmatrix} \binom{k}{0} a^k + \binom{k}{1} a^{k-1} b + \binom{k}{2} a^{k-2} b^2 + \cdots + \binom{k}{k-1} a b^{k-1} + \binom{k}{k} b^k \end{bmatrix} \text{ or } (a+b)^{k+1} = \binom{k}{0} a^{k+1} \\ + \binom{k}{0} a^k b + \binom{k}{1} a^k b + \binom{k}{1} a^{k-1} b^2 + \binom{k}{2} a^{k-1} b^2 + \binom{k}{2} a^{k-2} b^3 + \cdots + \binom{k}{k-1} a^2 b^{k-1} + \binom{k}{k-1} a b^k + \binom{k}{k} a b^k + \binom{k}{k} b^{k+1} \\ \mathbf{d.} \ (a+b)^{k+1} = \binom{k}{0} a^{k+1} + \begin{bmatrix} \binom{k}{0} + \binom{k}{1} \end{bmatrix} a^k b + \begin{bmatrix} \binom{k}{1} + \binom{k}{2} \end{bmatrix} a^{k-1} b^2 + \begin{bmatrix} \binom{k}{2} + \binom{k}{3} \end{bmatrix} a^{k-2} b^3 + \cdots + \begin{bmatrix} \binom{k}{k-1} + \binom{k}{k} \end{bmatrix} a b^k + \binom{k}{k} b^{k+1} \\ \mathbf{e.} \ (a+b)^{k+1} = \binom{k}{0} a^{k+1} + \binom{k+1}{1} a^k b + \binom{k+1}{2} a^{k-1} b^2 + \binom{k+1}{3} a^{k-2} b^3 + \cdots + \binom{k+1}{k} a b^k + \binom{k}{k} b^{k+1} \\ \mathbf{f.} \ (a+b)^{k+1} = \binom{k+1}{0} a^{k+1} + \binom{k+1}{1} a^k b + \binom{k+1}{2} a^{k-1} b^2 + \binom{k+1}{3} a^{k-2} b^3 + \cdots + \binom{k+1}{k} a b^k + \binom{k+1}{k+1} b^{k+1} \\ \mathbf{86.} \ 6840 \ \mathbf{87.} \ 56 \ \mathbf{88.} \ \text{true}$$

# Section 11.6

## **Check Point Exercises**

**1.** 72 **2.** 729 **3.** 676,000 **4.** 840 **5.** 720 6. a. combinations b. permutations 7. 210

# **Exercise Set 11.6**

**1.** 3024 **2.** 210 **3.** 6720 **4.** 5040 **5.** 720 **6.** 362,880 7. 1 8. 1 9. 126 10. 210 12. 792 11, 330 14. 1 15. 1 16. 1 17. combinations 18. permutations 19. permutations 20. combinations **25.** -9499 **26.** -2062 **27.**  $\frac{3}{68}$  **28.**  $\frac{21}{44}$  **29.** 27 ways **30.** 12 choices **31.** 40 ways **32.** 144 ways 33. 243 ways

34. 6561 ways 35. 144 area codes 36. 35,152 call letters 37. 120 ways 38. 24 ways 39. 6 paragraphs 40. 4 ways 41. 720 ways 42. 5040 ways 43. 8,648,640 ways 44. 6840 ways 45. 120 ways 46. 336 ways 47. 15,120 lineups

**48.** 840 arrangements **49.** 20 ways **50.** 330 committees **51.** 495 collections **52.** 3003 ways **53.** 24,310 groups **54.** about 3.07 × 10<sup>19</sup> ways **55.** 22,957,480 selections **56.** 45,057,474 selections **57.** 360 ways **58.** 76,904,685 selections

**59.** 1716 ways **60.** 177,600 ways **61.** 1140 ways **62.** 19,600 ways **63.** 840 passwords **64.** 15,120 ways **65.** 2730 cones **66.** 4495 bowls **67.** 720 **68.** 120 **69.** 20 **70.** 15 **71.** 24 **72.** 600 **83.** makes sense **84.** makes sense

85. does not make sense 86. does not make sense 87. false 88. false 89. true 90. false 91. 14,400 ways 92. 144 numbers

95.  $\frac{2}{3}$  96.  $\frac{1}{3}$  97.  $\frac{2}{3}$ 93. 450 ways

#### Section 11.7

### **Check Point Exercises**

**1. a.**  $\frac{7664}{100,000}$  or  $\frac{479}{6250} \approx 0.077$  **b.**  $\frac{720}{800}$  or  $\frac{9}{10} = 0.9$  **c.**  $\frac{720}{7664}$  or  $\frac{45}{479} \approx 0.094$  **2.**  $\frac{1}{3}$  **3.**  $\frac{1}{9}$  **4.**  $\frac{1}{13}$  **5.**  $\frac{1}{13,983,816} \approx 0.0000000715$ 6.  $\frac{160}{191}$  7.  $\frac{1}{3}$  8.  $\frac{3}{4}$  9. a. 0.99 b. 0.64 10.  $\frac{1}{361} \approx 0.00277$  11.  $\frac{1}{160}$ 

## Exercise Set 11.7

1. 0.10 2. 0.24 3. 0.52 4. 0.48 5. 0.01 6. 0.05 7. 0.59 8. 0.41 9. 0.61 10. 0.57 11.  $\frac{1}{6}$  12.  $\frac{1}{6}$  13. 14.  $\frac{1}{2}$  15.  $\frac{1}{3}$  16. 0 17.  $\frac{1}{13}$  18.  $\frac{1}{4}$  19.  $\frac{3}{13}$  20.  $\frac{3}{13}$  21.  $\frac{1}{4}$  22.  $\frac{1}{2}$  23.  $\frac{7}{8}$  24.  $\frac{1}{2}$  25.  $\frac{1}{12}$  26.  $\frac{5}{36}$  27.  $\frac{1}{18,009,460}$ ;  $\frac{5}{900,473}$  28.  $\frac{1}{593,775}$ ;  $\frac{4}{23,751}$  29. a. 2,598,960 b. 1287 c.  $\frac{1287}{2,598,960} \approx 0.0005$  30.  $\frac{11}{1105} \approx 0.00995$  31.  $\frac{4}{5}$  32.  $\frac{1}{6}$  33.  $\frac{50}{87}$  34.  $\frac{85}{174}$  35.  $\frac{113}{174}$  36.  $\frac{39}{58}$  37.  $\frac{12}{13}$  38.  $\frac{10}{13}$  39.  $\frac{2}{13}$  40.  $\frac{1}{13}$  41.  $\frac{7}{13}$  42.  $\frac{7}{13}$  43.  $\frac{3}{4}$  44.  $\frac{7}{8}$ **45.**  $\frac{33}{40}$  **46.**  $\frac{13}{20}$  **47.**  $\frac{1}{36}$  **48.**  $\frac{1}{36}$  **49.**  $\frac{1}{3}$  **50.**  $\frac{1}{6}$  **51.**  $\frac{1}{64}$  **52.**  $\frac{1}{128}$  **53.** a.  $\frac{1}{256}$  b.  $\frac{1}{4096}$  c.  $\left(\frac{15}{16}\right)^{10} \approx 0.524$ **d.**  $1 - \left(\frac{15}{16}\right)^{10} \approx 0.476$  **64.** does not make sense **65.** does not make sense **66.** does not make sense **67.** makes sense 71. a.  $\frac{12}{25}$  b.  $\frac{3}{10}$  72. 0.06 73. a. The first person can have any birthday in the year. The second person can have all but one birthday. **b.**  $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \approx 0.99$  **c.**  $\approx 0.01$  **d.**  $\approx 0.41$ 

### **Chapter 11 Review Exercises**

**1.**  $a_1 = 3$ ;  $a_2 = 10$ ;  $a_3 = 17$ ;  $a_4 = 24$  **2.**  $a_1 = -\frac{3}{2}$ ;  $a_2 = \frac{4}{3}$ ;  $a_3 = -\frac{5}{4}$ ;  $a_4 = \frac{6}{5}$  **3.**  $a_1 = 1$ ;  $a_2 = 1$ ;  $a_3 = \frac{1}{2}$ ;  $a_4 : \frac{1}{6}$ **4.**  $a_1 = \frac{1}{2}$ ;  $a_2 = -\frac{1}{4}$ ;  $a_3 = \frac{1}{8}$ ;  $a_4 = -\frac{1}{16}$  **5.**  $a_1 = 9$ ;  $a_2 = \frac{2}{27}$ ;  $a_3 = 9$ ;  $a_4 = \frac{2}{27}$  **6.**  $a_1 = 4$ ;  $a_2 = 11$ ;  $a_3 = 25$ ;  $a_4 = 53$  **7.** 65 **8.** 95 9. -20 **10.**  $\sum_{i=1}^{15} \frac{i}{i+2}$  **11.**  $\sum_{i=4}^{13} i^3$  or  $\sum_{i=1}^{10} (i+3)^3$  **12.** 7, 11, 15, 19, 23, 27 **13.** -4, -9, -14, -19, -24, -29 **14.**  $\frac{3}{2}$ , 1,  $\frac{1}{2}$ , 0,  $-\frac{1}{2}$ , -1 **15.** -2, 3, 8, 13, 18, 23 **16.**  $a_6 = 20$  **17.**  $a_{12} = -30$  **18.**  $a_{14} = -38$  **19.**  $a_n = 4n - 11; a_{20} = 69$  **20.**  $a_n = 220 - 20n; a_{20} = -180$  **21.**  $a_n = 8 - 5n; a_{20} = -92$  **22.** 1727 **23.** 225 **24.** 15,150 **25.** 440 **26.** -500 **27.** -2325 **28. a.**  $a_n = 4.75n + 34.25$  **b.** 96%