

## Technology

Graphing utilities have a menu item for calculating combinations, usually labeled  $nC_r$ . For example, to find  ${}_8C_3$ , the keystrokes on most graphing utilities are

$$8 \text{ [ } nC_r \text{ ] } 3 \text{ [ ENTER ]}$$

If you are using a scientific calculator, check your manual to see whether there is a menu item for calculating combinations.

If you use your calculator's factorial key to find  $\frac{8!}{5!3!}$ , be sure to enclose the factorials in the denominator with parentheses

$$8 \text{ [ ] } \div \text{ [ ( ] } 5 \text{ [ ] } \times \text{ [ ( ] } 3 \text{ [ ] } \text{ [ ) ]}$$

pressing  $\text{[ = ]}$  or  $\text{[ ENTER ]}$  to obtain the answer.


looking for the number of combinations of eight things taken three at a time. We use the formula

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

with  $n = 8$  and  $r = 3$ .

$${}_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$$

Thus, 56 committees of three people each can be formed from the eight people on the board of supervisors.

 **Check Point 7** From a group of 10 physicians, in how many ways can four people be selected to attend a conference on acupuncture?

### EXAMPLE 8 Using the Formula for Combinations

In poker, a person is dealt 5 cards from a standard 52-card deck. The order in which you are dealt the 5 cards does not matter. How many different 5-card poker hands are possible?

**Solution** Because the order in which the 5 cards are dealt does not matter, this is a problem involving combinations. We are looking for the number of combinations of  $n = 52$  cards dealt  $r = 5$  at a time. We use the formula

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

with  $n = 52$  and  $r = 5$ .


$${}_{52}C_5 = \frac{52!}{(52-5)!5!} = \frac{52!}{47!5!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47!}{47! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960$$

Thus, there are 2,598,960 different 5-card poker hands possible. It surprises many people that more than 2.5 million 5-card hands can be dealt from a mere 52 cards.

If you are a card player, it does not get any better than to be dealt the 5-card poker hand shown in **Figure 11.9**. This hand is called a *royal flush*. It consists of an ace, king, queen, jack, and 10, all of the same suit: all hearts, all diamonds, all clubs, or all spades. The probability of being dealt a royal flush involves calculating the number of ways of being dealt such a hand: just 4 of all 2,598,960 possible hands. In the next section, we move from counting possibilities to computing probabilities.



Figure 11.9 A royal flush

 **Check Point 8** How many different 4-card hands can be dealt from a deck that has 16 different cards?

## Exercise Set 11.6

### Practice Exercises

In Exercises 1–8, use the formula for  ${}_nP_r$  to evaluate each expression.

- ${}_9P_4$
- ${}_7P_3$
- ${}_8P_5$
- ${}_{10}P_4$
- ${}_6P_6$
- ${}_9P_9$
- ${}_8P_0$
- ${}_6P_0$

In Exercises 9–16, use the formula for  ${}_nC_r$  to evaluate each expression.

- ${}_9C_5$
- ${}_{10}C_6$
- ${}_{11}C_4$
- ${}_{12}C_5$
- ${}_7C_7$
- ${}_4C_4$
- ${}_5C_0$
- ${}_6C_0$



In Exercises 17–20, does the problem involve permutations or combinations? Explain your answer. (It is not necessary to solve the problem.)

17. A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?
18. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If first prize is \$1000, second prize is \$500, and third prize is \$100, in how many different ways can the prizes be awarded?
19. How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?
20. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If each prize is \$500, in how many different ways can the prizes be awarded?
34. You are taking a multiple-choice test that has eight questions. Each of the questions has three answer choices, with one correct answer per question. If you select one of these three choices for each question and leave nothing blank, in how many ways can you answer the questions?
35. In the original plan for area codes in 1945, the first digit could be any number from 2 through 9, the second digit was either 0 or 1, and the third digit could be any number except 0. With this plan, how many different area codes were possible?
36. How many different four-letter radio station call letters can be formed if the first letter must be W or K?
37. Six performers are to present their comedy acts on a weekend evening at a comedy club. One of the performers insists on being the last stand-up comic of the evening. If this performer's request is granted, how many different ways are there to schedule the appearances?
38. Five singers are to perform at a night club. One of the singers insists on being the last performer of the evening. If this singer's request is granted, how many different ways are there to schedule the appearances?
39. In the *Cambridge Encyclopedia of Language* (Cambridge University Press, 1987), author David Crystal presents five sentences that make a reasonable paragraph regardless of their order. The sentences are as follows:
  - Mark had told him about the foxes.
  - John looked out the window.
  - Could it be a fox?
  - However, nobody had seen one for months.
  - He thought he saw a shape in the bushes.

## Practice Plus

In Exercises 21–28, evaluate each expression.

21.  $\frac{7P_3}{3!} - 7C_3$
22.  $\frac{20P_2}{2!} - 20C_2$
23.  $1 - \frac{3P_2}{4P_3}$
24.  $1 - \frac{5P_3}{10P_4}$
25.  $\frac{7C_3}{5C_4} - \frac{98!}{96!}$
26.  $\frac{10C_3}{6C_4} - \frac{46!}{44!}$
27.  $\frac{4C_2 \cdot 6C_1}{18C_3}$
28.  $\frac{5C_1 \cdot 7C_2}{12C_3}$

## Application Exercises

Use the Fundamental Counting Principle to solve Exercises 29–40.

29. The model of the car you are thinking of buying is available in nine different colors and three different styles (hatchback, sedan, or station wagon). In how many ways can you order the car?
30. A popular brand of pen is available in three colors (red, green, or blue) and four writing tips (bold, medium, fine, or micro). How many different choices of pens do you have with this brand?
31. An ice cream store sells two drinks (sodas or milk shakes), in four sizes (small, medium, large, or jumbo), and five flavors (vanilla, strawberry, chocolate, coffee, or pistachio). In how many ways can a customer order a drink?
32. A restaurant offers the following lunch menu.
 

Main Course	Vegetables	Beverages	Desserts
Ham	Potatoes	Coffee	Cake
Chicken	Peas	Tea	Pie
Fish	Green beans	Milk	Ice cream
Beef		Soda	

If one item is selected from each of the four groups, in how many ways can a meal be ordered? Describe two such orders.

33. You are taking a multiple-choice test that has five questions. Each of the questions has three answer choices, with one correct answer per question. If you select one of these three choices for each question and leave nothing blank, in how many ways can you answer the questions?
40. A television programmer is arranging the order that five movies will be seen between the hours of 6 P.M. and 4 A.M. Two of the movies have a G rating and they are to be shown in the first two time blocks. One of the movies is rated NC-17 and it is to be shown in the last of the time blocks, from 2 A.M. until 4 A.M. Given these restrictions, in how many ways can the five movies be arranged during the indicated time blocks?

Use the formula for  ${}_nP_r$  to solve Exercises 41–48.

41. A club with ten members is to choose three officers—president, vice-president, and secretary-treasurer. If each office is to be held by one person and no person can hold more than one office, in how many ways can those offices be filled?
42. A corporation has ten members on its board of directors. In how many different ways can it elect a president, vice-president, secretary, and treasurer?
43. For a segment of a radio show, a disc jockey can play 7 songs. If there are 13 songs to select from, in how many ways can the program for this segment be arranged?
44. Suppose you are asked to list, in order of preference, the three best movies you have seen this year. If you saw 20 movies during the year, in how many ways can the three best be chosen and ranked?



45. In a race in which six automobiles are entered and there are no ties, in how many ways can the first three finishers come in?
46. In a production of *West Side Story*, eight actors are considered for the male roles of Tony, Riff, and Bernardo. In how many ways can the director cast the male roles?
47. Nine bands have volunteered to perform at a benefit concert, but there is only enough time for five of the bands to play. How many lineups are possible?
48. How many arrangements can be made using four of the letters of the word COMBINE if no letter is to be used more than once?

Use the formula for  ${}_nC_r$  to solve Exercises 49–56.

49. An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?
50. A four-person committee is to be elected from an organization's membership of 11 people. How many different committees are possible?
51. Of 12 possible books, you plan to take 4 with you on vacation. How many different collections of 4 books can you take?
52. There are 14 standbys who hope to get seats on a flight, but only 6 seats are available on the plane. How many different ways can the 6 people be selected?
53. You volunteer to help drive children at a charity event to the zoo, but you can fit only 8 of the 17 children present in your van. How many different groups of 8 children can you drive?
54. Of the 100 people in the U.S. Senate, 18 serve on the Foreign Relations Committee. How many ways are there to select Senate members for this committee (assuming party affiliation is not a factor in selection)?
55. To win at LOTTO in the state of Florida, one must correctly select 6 numbers from a collection of 53 numbers (1 through 53). The order in which the selection is made does not matter. How many different selections are possible?
56. To win in the New York State lottery, one must correctly select 6 numbers from 59 numbers. The order in which the selection is made does not matter. How many different selections are possible?

In Exercises 57–66, solve by the method of your choice.

57. In a race in which six automobiles are entered and there are no ties, in how many ways can the first four finishers come in?
58. A book club offers a choice of 8 books from a list of 40. In how many ways can a member make a selection?
59. A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?
60. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If first prize is \$1000, second prize is \$500, and third prize is \$100, in how many different ways can the prizes be awarded?
61. From a club of 20 people, in how many ways can a group of three members be selected to attend a conference?
62. Fifty people purchase raffle tickets. Three winning tickets are selected at random. If each prize is \$500, in how many different ways can the prizes be awarded?

63. How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?
64. Nine comedy acts will perform over two evenings. Five of the acts will perform on the first evening and the order in which the acts perform is important. How many ways can the schedule for the first evening be made?
65. Using 15 flavors of ice cream, how many cones with three different flavors can you create if it is important to you which flavor goes on the top, middle, and bottom?
66. Baskin-Robbins offers 31 different flavors of ice cream. One of their items is a bowl consisting of three scoops of ice cream, each a different flavor. How many such bowls are possible?

Exercises 67–72 are based on the following jokes about books:

- “Outside of a dog, a book is man’s best friend. Inside of a dog, it’s too dark to read.”—Groucho Marx
- “I recently bought a book of free verse. For \$12.”—George Carlin
- “If a word in the dictionary was misspelled, how would we know?”—Steven Wright
- “Encyclopedia is a Latin term. It means ‘to paraphrase a term paper.’”—Greg Ray
- “A bookstore is one of the only pieces of evidence we have that people are still thinking.”—Jerry Seinfeld
- “I honestly believe there is absolutely nothing like going to bed with a good book. Or a friend who’s read one.”—Phyllis Diller

67. In how many ways can these six jokes be ranked from best to worst?
68. If Phyllis Diller’s joke about books is excluded, in how many ways can the remaining five jokes be ranked from best to worst?
69. In how many ways can people select their three favorite jokes from these comments about books?
70. In how many ways can people select their two favorite jokes from these comments about books?
71. If the order in which these jokes are told makes a difference in terms of how they are received, how many ways can they be delivered if George Carlin’s joke is delivered first and Jerry Seinfeld’s joke is told last?
72. If the order in which these jokes are told makes a difference in terms of how they are received, how many ways can they be delivered if a joke by a man is told first?

## Writing in Mathematics

73. Explain the Fundamental Counting Principle.
74. Write an original problem that can be solved using the Fundamental Counting Principle. Then solve the problem.
75. What is a permutation?
76. Describe what  ${}_nP_r$  represents.
77. Write a word problem that can be solved by evaluating  ${}_7P_3$ .
78. What is a combination?
79. Explain how to distinguish between permutation and combination problems.
80. Write a word problem that can be solved by evaluating  ${}_7C_3$ .