28. S_1: 3 is a factor of 0; S_k: 3 is a factor of $k(k+1)(k-1)$ or $k^3 - k$; S_{k+1}: 3 is a factor of $(k+1)(k+2)k$ or $k^3 + 3k^2 + 2k$; S_{k+1} can be obtained from S_k by rewriting $k^3 + 3k^2 + 2k$ as $(k^3 - k) + 3(k^2 + k)$.

29. S_1: $5 \cdot 6^1 = 6(6^1 - 1)$; S_k: $\sum_{i=1}^{k} 5 \cdot 6^i = 6(6^k - 1)$; S_{k+1}: $\sum_{i=1}^{k+1} 5 \cdot 6^i = 6(6^{k+1} - 1)$; S_{k+1} can be obtained by adding $5 \cdot 6^{k+1}$ to both sides of S_k.

30. S_1: $7 \cdot 8^1 = 8(8^1 - 1)$; S_k: $\sum_{i=1}^{k} 7 \cdot 8^i = 8(8^k - 1)$; S_{k+1}: $\sum_{i=1}^{k+1} 7 \cdot 8^i = 8(8^{k+1} - 1)$; S_{k+1} can be obtained by adding $7 \cdot 8^{k+1}$ to both sides of S_k.

31. S_1: $1 + 2 > 1$; S_k: $k + 2 > k$; S_{k+1}: $k + 3 > k + 1$; S_{k+1} can be obtained by adding 1 to both sides of S_k.

32. S_1: If $0 < x < 1$, then $0 < x^1 < 1$; S_k: If $0 < x < 1$, then $0 < x^k < 1$; S_{k+1}: If $0 < x < 1$, then $0 < x^{k+1} < 1$; S_{k+1} can be obtained by multiplying the respective parts of $0 < x < 1$ and $0 < x^k < 1$.

33. S_1: $(ab)^1 = a^1b^1$; S_k: $(ab)^k = a^kb^k$; S_{k+1}: $(ab)^{k+1} = a^{k+1}b^{k+1}$; S_{k+1} can be obtained by multiplying both sides of S_k by (ab).

34. S_1: $\left(\frac{a}{b}\right)^1 = \frac{a^1}{b^1}$; S_k: $\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$; S_{k+1}: $\left(\frac{a}{b}\right)^{k+1} = \frac{a^{k+1}}{b^{k+1}}$; S_{k+1} can be obtained by multiplying both sides of S_k by $\left(\frac{a}{b}\right)$.

37. does not make sense **38.** does not make sense **39.** does not make sense **40.** makes sense

41. S_3: $3^2 > 2(3) + 1$; S_k: $k^2 > 2k + 1$ for $k \geq 3$; S_{k+1}: $(k+1)^2 > 2(k+1) + 1$ or $k^2 + 2k + 1 > 2k + 3$; S_{k+1} can be obtained from S_k by noting that S_{k+1} is the same as $k^2 > 2$ which is true for $k \geq 3$.

42. S_5: $2^5 > 5^2$; S_k: $2^k > k^2$ for $k \geq 5$; S_{k+1}: $2^{k+1} > (k+1)^2$ or $2(2^k) > k^2 + 2k + 1$; S_{k+1} can be obtained from S_k by multiplying both sides of S_k by 2 and noting that $k^2 > 2k + 1$ for $k \geq 5$.

43. S_1: $\frac{1}{4}$; S_2: $\frac{1}{3}$; S_3: $\frac{3}{8}$; S_4: $\frac{2}{5}$; S_5: $\frac{5}{12}$; S_n: $\frac{n}{2n+2}$; Use S_k to obtain the conjectured formula.

44. S_1: $\frac{1}{2}$; S_2: $\frac{1}{3}$; S_3: $\frac{1}{4}$; S_4: $\frac{1}{5}$; S_5: $\frac{1}{6}$; S_n: $\frac{1}{n+1}$; Use S_k to obtain the conjectured formula.

46. The exponents begin with the exponent on $a + b$ and decrease by 1 in each successive term. **47.** The exponents begin with 0, increase by 1 in each successive term, and end with the exponent on $a + b$. **48.** The sum of the exponents is the exponent on $a + b$.

Section 11.5

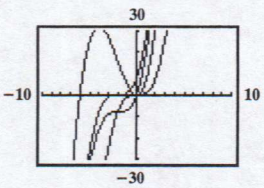
Check Point Exercises

1. a. 20 **b.** 1 **c.** 28 **d.** 1 **2.** $x^4 + 4x^3 + 6x^2 + 4x + 1$ **3.** $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$ **4.** $4032x^5y^4$

Exercise Set 11.5

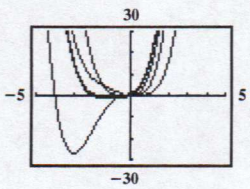
1. 56 **2.** 21 **3.** 12 **4.** 11 **5.** 1 **6.** 105 **7.** 4950 **8.** 4950 **9.** $x^3 + 6x^2 + 12x + 8$ **10.** $x^3 + 12x^2 + 48x + 64$
11. $27x^3 + 27x^2y + 9xy^2 + y^3$ **12.** $x^3 + 9x^2y + 27xy^2 + 27y^3$ **13.** $125x^3 - 75x^2 + 15x - 1$ **14.** $64x^3 - 48x^2 + 12x - 1$
15. $16x^4 + 32x^3 + 24x^2 + 8x + 1$ **16.** $81x^4 + 108x^3 + 54x^2 + 12x + 1$ **17.** $x^8 + 8x^6y + 24x^4y^2 + 32x^2y^3 + 16y^4$
18. $x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$ **19.** $y^4 - 12y^3 + 54y^2 - 108y + 81$ **20.** $y^4 - 16y^3 + 96y^2 - 256y + 256$
21. $16x^{12} - 32x^9 + 24x^6 - 8x^3 + 1$ **22.** $16x^{20} - 32x^{15} + 24x^{10} - 8x^5 + 1$ **23.** $c^5 + 10c^4 + 40c^3 + 80c^2 + 80c + 32$
24. $c^5 + 15c^4 + 90c^3 + 270c^2 + 405c + 243$ **25.** $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$ **26.** $x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$
27. $243x^5 - 405x^4y + 270x^3y^2 - 90x^2y^3 + 15xy^4 - y^5$ **28.** $x^5 - 15x^4y + 90x^3y^2 - 270x^2y^3 + 405xy^4 - 243y^5$
29. $64a^6 + 192a^5b + 240a^4b^2 + 160a^3b^3 + 60a^2b^4 + 12ab^5 + b^6$ **30.** $a^6 + 12a^5b + 60a^4b^2 + 160a^3b^3 + 240a^2b^4 + 192ab^5 + 64b^6$
31. $x^8 + 16x^7 + 112x^6 + \cdots$ **32.** $x^8 + 24x^7 + 252x^6 + \cdots$ **33.** $x^{10} - 20x^9y + 180x^8y^2 - \cdots$ **34.** $x^9 - 18x^8y + 144x^7y^2 - \cdots$
35. $x^{32} + 16x^{30} + 120x^{28} + \cdots$ **36.** $x^{34} + 17x^{32} + 136x^{30} + \cdots$ **37.** $y^{60} - 20y^{57} + 190y^{54} - \cdots$ **38.** $y^{63} - 21y^{60} + 210y^{57} - \cdots$
39. $240x^4y^2$ **40.** $60x^4y^2$ **41.** $126x^5$ **42.** $210x^6$ **43.** $56x^6y^{15}$ **44.** $56x^9y^{10}$ **45.** $-\frac{21}{2}x^6$ **46.** $7x^5$ **47.** $319{,}770x^{16}y^{14}$
48. $13{,}440x^4y^6$ **49.** $x^{12} + 4x^7 + 6x^2 + \frac{4}{x^3} + \frac{1}{x^8}$ **50.** $x^8 + 4x^3 + \frac{6}{x^2} + \frac{4}{x^7} + \frac{1}{x^{12}}$ **51.** $x - 3x^{1/3} + \frac{3}{x^{1/3}} - \frac{1}{x}$ **52.** $x^2 - 3x + 3 - \frac{1}{x}$
53. $4x^3 + 6x^2h + 4xh^2 + h^3$ **54.** $5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$ **55.** 252 **56.** $924x^6$ **57.** 0.1138 **58.** 0.0134

68.



f_2, f_3, and f_4 are approaching $f_1 = f_5$.

69.



f_2, f_3, f_4, and f_5 are approaching $f_1 = f_6$.

70. $f_1(x) = x^3 - 3x^2 + 3x - 1$
71. $f_1(x) = x^4 - 8x^3 + 24x^2 - 32x + 16$
72. $f_1(x) = x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$
73. makes sense **74.** makes sense
75. does not make sense **76.** does not make sense
77. false **78.** true
79. false **80.** false

81. $x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1$ **82.** $10x^4y^6$ **83.** $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = \frac{n!}{(n-r)![n-(n-r)]!} = \binom{n}{n-r}$

84. $\binom{n}{r} + \binom{n}{r+1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-(r+1))!} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)r!(n-r-1)!} = \frac{n!(r+1) + n!(n-r)}{(r+1)r!(n-r)!}$
$= \frac{n!(r+1+n-r)}{(r+1)r!(n-r)!} = \frac{n!(n+1)}{(r+1)r!(n-r)!} = \frac{(n+1)!}{(r+1)!(n-r)!} = \frac{(n+1)!}{(r+1)!(n+1-(r+1))!} = \binom{n+1}{r+1}$

85. a. $(a+b)^1 = a + b = \binom{1}{0}a^1 + \binom{1}{1}b^1$ **b.** Assume: $(a+b)^k = \binom{k}{0}a^k + \binom{k}{1}a^{k-1}b + \binom{k}{2}a^{k-2}b^2 + \cdots + \binom{k}{k-1}ab^{k-1}$
$+ \binom{k}{k}b^k$; Prove: $(a+b)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^{k+1-1}b + \binom{k+1}{2}a^{k+1-2}b^2 + \cdots + \binom{k+1}{k+1-1}ab^{k+1-1} + \binom{k+1}{k+1}b^{k+1}$

c. $(a+b)(a+b)^k = (a+b)\left[\binom{k}{0}a^k + \binom{k}{1}a^{k-1}b + \binom{k}{2}a^{k-2}b^2 + \cdots + \binom{k}{k-1}ab^{k-1} + \binom{k}{k}b^k\right]$ or $(a+b)^{k+1} = \binom{k}{0}a^{k+1}$
$+ \binom{k}{0}a^kb + \binom{k}{1}a^kb + \binom{k}{1}a^{k-1}b^2 + \binom{k}{2}a^{k-1}b^2 + \binom{k}{2}a^{k-2}b^3 + \cdots + \binom{k}{k-1}a^2b^{k-1} + \binom{k}{k-1}ab^k + \binom{k}{k}ab^k + \binom{k}{k}b^{k+1}$

d. $(a+b)^{k+1} = \binom{k}{0}a^{k+1} + \left[\binom{k}{0} + \binom{k}{1}\right]a^kb + \left[\binom{k}{1} + \binom{k}{2}\right]a^{k-1}b^2 + \left[\binom{k}{2} + \binom{k}{3}\right]a^{k-2}b^3 + \cdots + \left[\binom{k}{k-1} + \binom{k}{k}\right]ab^k + \binom{k}{k}b^{k+1}$

e. $(a+b)^{k+1} = \binom{k}{0}a^{k+1} + \binom{k+1}{1}a^kb + \binom{k+1}{2}a^{k-1}b^2 + \binom{k+1}{3}a^{k-2}b^3 + \cdots + \binom{k+1}{k}ab^k + \binom{k}{k}b^{k+1}$

f. $(a+b)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^kb + \binom{k+1}{2}a^{k-1}b^2 + \binom{k+1}{3}a^{k-2}b^3 + \cdots + \binom{k+1}{k}ab^k + \binom{k+1}{k+1}b^{k+1}$

86. 6840 **87.** 56 **88.** true

Section 11.6

Check Point Exercises

1. 72 **2.** 729 **3.** 676,000 **4.** 840 **5.** 720 **6. a.** combinations **b.** permutations **7.** 210 **8.** 1820

Exercise Set 11.6

1. 3024 **2.** 210 **3.** 6720 **4.** 5040 **5.** 720 **6.** 362,880 **7.** 1 **8.** 1 **9.** 126 **10.** 210 **11.** 330 **12.** 792 **13.** 1
14. 1 **15.** 1 **16.** 1 **17.** combinations **18.** permutations **19.** permutations **20.** combinations **21.** 0 **22.** 0 **23.** $\frac{3}{4}$
24. $\frac{83}{84}$ **25.** -9499 **26.** -2062 **27.** $\frac{3}{68}$ **28.** $\frac{21}{44}$ **29.** 27 ways **30.** 12 choices **31.** 40 ways **32.** 144 ways **33.** 243 ways
34. 6561 ways **35.** 144 area codes **36.** 35,152 call letters **37.** 120 ways **38.** 24 ways **39.** 6 paragraphs **40.** 4 ways
41. 720 ways **42.** 5040 ways **43.** 8,648,640 ways **44.** 6840 ways **45.** 120 ways **46.** 336 ways **47.** 15,120 lineups
48. 840 arrangements **49.** 20 ways **50.** 330 committees **51.** 495 collections **52.** 3003 ways **53.** 24,310 groups
54. about 3.07×10^{19} ways **55.** 22,957,480 selections **56.** 45,057,474 selections **57.** 360 ways **58.** 76,904,685 selections
59. 1716 ways **60.** 177,600 ways **61.** 1140 ways **62.** 19,600 ways **63.** 840 passwords **64.** 15,120 ways **65.** 2730 cones
66. 4495 bowls **67.** 720 **68.** 120 **69.** 20 **70.** 15 **71.** 24 **72.** 600 **83.** makes sense **84.** makes sense
85. does not make sense **86.** does not make sense **87.** false **88.** false **89.** true **90.** false **91.** 14,400 ways **92.** 144 numbers
93. 450 ways **95.** $\frac{2}{3}$ **96.** $\frac{1}{3}$ **97.** $\frac{2}{3}$

Section 11.7

Check Point Exercises

1. a. $\frac{7664}{100{,}000}$ or $\frac{479}{6250} \approx 0.077$ **b.** $\frac{720}{800}$ or $\frac{9}{10} = 0.9$ **c.** $\frac{720}{7664}$ or $\frac{45}{479} \approx 0.094$ **2.** $\frac{1}{3}$ **3.** $\frac{1}{9}$ **4.** $\frac{1}{13}$ **5.** $\frac{1}{13{,}983{,}816} \approx 0.0000000715$
6. $\frac{160}{191}$ **7.** $\frac{1}{3}$ **8.** $\frac{3}{4}$ **9. a.** 0.99 **b.** 0.64 **10.** $\frac{1}{361} \approx 0.00277$ **11.** $\frac{1}{16}$

Exercise Set 11.7

1. 0.10 **2.** 0.24 **3.** 0.52 **4.** 0.48 **5.** 0.01 **6.** 0.05 **7.** 0.59 **8.** 0.41 **9.** 0.61 **10.** 0.57 **11.** $\frac{1}{6}$ **12.** $\frac{1}{6}$ **13.** $\frac{1}{2}$
14. $\frac{1}{2}$ **15.** $\frac{1}{3}$ **16.** 0 **17.** $\frac{1}{13}$ **18.** $\frac{1}{4}$ **19.** $\frac{3}{13}$ **20.** $\frac{3}{13}$ **21.** $\frac{1}{4}$ **22.** $\frac{1}{2}$ **23.** $\frac{7}{8}$ **24.** $\frac{1}{2}$ **25.** $\frac{1}{12}$ **26.** $\frac{5}{36}$
27. $\frac{1}{18{,}009{,}460}; \frac{5}{900{,}473}$ **28.** $\frac{1}{593{,}775}; \frac{4}{23{,}751}$ **29. a.** 2,598,960 **b.** 1287 **c.** $\frac{1287}{2{,}598{,}960} \approx 0.0005$ **30.** $\frac{11}{1105} \approx 0.00995$ **31.** $\frac{43}{58}$
32. $\frac{1}{6}$ **33.** $\frac{50}{87}$ **34.** $\frac{85}{174}$ **35.** $\frac{113}{174}$ **36.** $\frac{39}{58}$ **37.** $\frac{12}{13}$ **38.** $\frac{10}{13}$ **39.** $\frac{2}{13}$ **40.** $\frac{1}{13}$ **41.** $\frac{7}{13}$ **42.** $\frac{7}{13}$ **43.** $\frac{3}{4}$ **44.** $\frac{7}{8}$
45. $\frac{33}{40}$ **46.** $\frac{13}{20}$ **47.** $\frac{1}{36}$ **48.** $\frac{1}{36}$ **49.** $\frac{1}{3}$ **50.** $\frac{1}{6}$ **51.** $\frac{1}{64}$ **52.** $\frac{1}{128}$ **53. a.** $\frac{1}{256}$ **b.** $\frac{1}{4096}$ **c.** $\left(\frac{15}{16}\right)^{10} \approx 0.524$
d. $1 - \left(\frac{15}{16}\right)^{10} \approx 0.476$ **64.** does not make sense **65.** does not make sense **66.** does not make sense **67.** makes sense **68.** $\frac{3}{8}$
70. $\frac{1}{10}$ **71. a.** $\frac{12}{25}$ **b.** $\frac{3}{10}$ **72.** 0.06 **73. a.** The first person can have any birthday in the year. The second person can have all but one birthday. **b.** $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \approx 0.99$ **c.** ≈ 0.01 **d.** ≈ 0.41 **e.** 23 people

Chapter 11 Review Exercises

1. $a_1 = 3; a_2 = 10; a_3 = 17; a_4 = 24$ **2.** $a_1 = -\frac{3}{2}; a_2 = \frac{4}{3}; a_3 = -\frac{5}{4}; a_4 = \frac{6}{5}$ **3.** $a_1 = 1; a_2 = 1; a_3 = \frac{1}{2}; a_4: \frac{1}{6}$
4. $a_1 = \frac{1}{2}; a_2 = -\frac{1}{4}; a_3 = \frac{1}{8}; a_4 = -\frac{1}{16}$ **5.** $a_1 = 9; a_2 = \frac{2}{27}; a_3 = 9; a_4 = \frac{2}{27}$ **6.** $a_1 = 4; a_2 = 11; a_3 = 25; a_4 = 53$ **7.** 65 **8.** 95
9. -20 **10.** $\sum_{i=1}^{15} \frac{i}{i+2}$ **11.** $\sum_{i=4}^{13} i^3$ or $\sum_{i=1}^{10} (i+3)^3$ **12.** 7, 11, 15, 19, 23, 27 **13.** $-4, -9, -14, -19, -24, -29$ **14.** $\frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1$
15. $-2, 3, 8, 13, 18, 23$ **16.** $a_6 = 20$ **17.** $a_{12} = -30$ **18.** $a_{14} = -38$ **19.** $a_n = 4n - 11; a_{20} = 69$ **20.** $a_n = 220 - 20n; a_{20} = -180$
21. $a_n = 8 - 5n; a_{20} = -92$ **22.** 1727 **23.** 225 **24.** 15,150 **25.** 440 **26.** -500 **27.** -2325 **28. a.** $a_n = 4.75n + 34.25$ **b.** 96%