Exercise Set 11.3

Practice Exercises

In Exercises 1–8, write the first five terms of each geometric sequence.

1. $a_1 = 5$, $r = 3$

2. $a_1 = 4$, $r = 3$

3. $a_1 = 20$, $r = \frac{1}{2}$

4. $a_1 = 24$, $r = \frac{1}{3}$

5. $a_n = -4a_{n-1}$, $a_1 = 10$

6. $a_n = -3a_{n-1}$, $a_1 = 10$

7. $a_n = -5a_{n-1}$, $a_1 = -6$

8. $a_n = -6a_{n-1}$, $a_1 = -2$

In Exercises 9–16, use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of each sequence with the given first term, a_1, and common ratio, r.

9. Find a_8 when $a_1 = 6$, $r = 2$.

10. Find a_8 when $a_1 = 5$, $r = 3$.

11. Find a_{12} when $a_1 = 5$, $r = -2$.

12. Find a_{12} when $a_1 = 4$, $r = -2$.

13. Find a_{40} when $a_1 = 1000$, $r = -\frac{1}{2}$.

14. Find a_{30} when $a_1 = 8000$, $r = -\frac{1}{2}$.

15. Find a_8 when $a_1 = 1{,}000{,}000$, $r = 0.1$.

16. Find a_8 when $a_1 = 40{,}000$, $r = 0.1$.

In Exercises 17–24, write a formula for the general term (the nth term) of each geometric sequence. Then use the formula for a_n to find a_7, the seventh term of the sequence.

17. $3, 12, 48, 192, \ldots$

18. $3, 15, 75, 375, \ldots$

19. $18, 6, 2, \frac{2}{3}, \ldots.$

20. $12, 6, 3, \frac{3}{2}, \ldots.$

21. $1.5, -3, 6, -12, \ldots$

22. $5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots.$

23. $0.0004, -0.004, 0.04, -0.4, \ldots$

24. $0.0007, -0.007, 0.07, -0.7, \ldots$

Use the formula for the sum of the first n terms of a geometric sequence to solve Exercises 25–30.

25. Find the sum of the first 12 terms of the geometric sequence: $2, 6, 18, 54, \ldots.$

26. Find the sum of the first 12 terms of the geometric sequence: $3, 6, 12, 24, \ldots.$

27. Find the sum of the first 11 terms of the geometric sequence: $3, -6, 12, -24, \ldots.$

28. Find the sum of the first 11 terms of the geometric sequence: $4, -12, 36, -108, \ldots.$

29. Find the sum of the first 14 terms of the geometric sequence: $-\frac{3}{2}, 3, -6, 12, \ldots.$

30. Find the sum of the first 14 terms of the geometric sequence: $-\frac{1}{24}, \frac{1}{12}, -\frac{1}{6}, \frac{1}{3}, \ldots.$

In Exercises 31–36, find the indicated sum. Use the formula for the sum of the first n terms of a geometric sequence.

31. $\sum_{i=1}^{8} 3^i$

32. $\sum_{i=1}^{6} 4^i$

33. $\sum_{i=1}^{10} 5 \cdot 2^i$

34. $\sum_{i=1}^{7} 4(-3)^i$

35. $\sum_{i=1}^{6} \left(\frac{1}{2}\right)^{i+1}$

36. $\sum_{i=1}^{6} \left(\frac{1}{3}\right)^{i+1}$

In Exercises 37–44, find the sum of each infinite geometric series.

37. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

38. $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

39. $3 + \frac{3}{4} + \frac{3}{4^2} + \frac{3}{4^3} + \cdots$

40. $5 + \frac{5}{6} + \frac{5}{6^2} + \frac{5}{6^3} + \cdots$

41. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$

42. $3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots$

43. $\sum_{i=1}^{\infty} 8(-0.3)^{i-1}$

44. $\sum_{i=1}^{\infty} 12(-0.7)^{i-1}$

In Exercises 45–50, express each repeating decimal as a fraction in lowest terms.

45. $0.\overline{5} = \frac{5}{10} + \frac{5}{100} + \frac{5}{1000} + \frac{5}{10{,}000} + \cdots$

46. $0.\overline{1} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10{,}000} + \cdots$

47. $0.\overline{47} = \frac{47}{100} + \frac{47}{10{,}000} + \frac{47}{1{,}000{,}000} + \cdots$

48. $0.\overline{83} = \frac{83}{100} + \frac{83}{10{,}000} + \frac{83}{1{,}000{,}000} + \cdots$

49. $0.\overline{257}$

50. $0.\overline{529}$

In Exercises 51–56, the general term of a sequence is given. Determine whether the sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference; if it is geometric, find the common ratio.

51. $a_n = n + 5$

52. $a_n = n - 3$

53. $a_n = 2^n$

54. $a_n = \left(\frac{1}{2}\right)^n$

55. $a_n = n^2 + 5$

56. $a_n = n^2 - 3$

Practice Plus

In Exercises 57–62, let

$$\{a_n\} = -5, 10, -20, 40, \ldots,$$

$$\{b_n\} = 10, -5, -20, -35, \ldots,$$

and

$$\{c_n\} = -2, 1, -\frac{1}{2}, \frac{1}{4}, \ldots.$$

57. Find $a_{10} + b_{10}$.

58. Find $a_{11} + b_{11}$.

59. Find the difference between the sum of the first 10 terms of $\{a_n\}$ and the sum of the first 10 terms of $\{b_n\}$.

60. Find the difference between the sum of the first 11 terms of $\{a_n\}$ and the sum of the first 11 terms of $\{b_n\}$.

61. Find the product of the sum of the first 6 terms of $\{a_n\}$ and the sum of the infinite series containing all the terms of $\{c_n\}$.

62. Find the product of the sum of the first 9 terms of $\{a_n\}$ and the sum of the infinite series containing all the terms of $\{c_n\}$.

In Exercises 63–64, find a_2 and a_3 for each geometric sequence.

63. $8, a_2, a_3, 27$

64. $2, a_2, a_3, -54$

Application Exercises

Use the formula for the general term (the nth term) of a geometric sequence to solve Exercises 65–68.

In Exercises 65–66, suppose you save $1 the first day of a month, $2 the second day, $4 the third day, and so on. That is, each day you save twice as much as you did the day before.

65. What will you put aside for savings on the fifteenth day of the month?

66. What will you put aside for savings on the thirtieth day of the month?

67. A professional baseball player signs a contract with a beginning salary of $3,000,000 for the first year and an annual increase of 4% per year beginning in the second year. That is, beginning in year 2, the athlete's salary will be 1.04 times what it was in the previous year. What is the athlete's salary for year 7 of the contract? Round to the nearest dollar.

68. You are offered a job that pays $30,000 for the first year with an annual increase of 5% per year beginning in the second year. That is, beginning in year 2, your salary will be 1.05 times what it was in the previous year. What can you expect to earn in your sixth year on the job?

In Exercises 69–70, you will develop geometric sequences that model the population growth for California and Texas, the two most-populated U.S. states.

69. The table shows population estimates for California from 2003 through 2006 from the U.S. Census Bureau.

Year	2003	2004	2005	2006
Population in millions	35.48	35.89	36.13	36.46

    a. Divide the population for each year by the population in the preceding year. Round to two decimal places and show that California has a population increase that is approximately geometric.

    b. Write the general term of the geometric sequence modeling California's population, in millions, *n* years after 2002.

    c. Use your model from part (b) to project California's population, in millions, for the year 2010. Round to two decimal places.

70. The table shows population estimates for Texas from 2003 through 2006 from the U.S. Census Bureau.

Year	2003	2004	2005	2006
Population in millions	22.12	22.49	22.86	23.41

    a. Divide the population for each year by the population in the preceding year. Round to two decimal places and show that Texas has a population increase that is approximately geometric.

    b. Write the general term of the geometric sequence modeling Texas's population, in millions, *n* years after 2002.

    c. Use your model from part (b) to project Texas's population, in millions, for the year 2010. Round to two decimal places.

Use the formula for the sum of the first n terms of a geometric sequence to solve Exercises 71–76.

In Exercises 71–72, you save $1 the first day of a month, $2 the second day, $4 the third day, continuing to double your savings each day.

71. What will your total savings be for the first 15 days?

72. What will your total savings be for the first 30 days?

73. A job pays a salary of $24,000 the first year. During the next 19 years, the salary increases by 5% each year. What is the total lifetime salary over the 20-year period? Round to the nearest dollar.

74. You are investigating two employment opportunities. Company A offers $30,000 the first year. During the next four years, the salary is guaranteed to increase by 6% per year. Company B offers $32,000 the first year, with guaranteed annual increases of 3% per year after that. Which company offers the better total salary for a five-year contract? By how much? Round to the nearest dollar.

75. A pendulum swings through an arc of 20 inches. On each successive swing, the length of the arc is 90% of the previous length.

$$20, \quad 0.9(20), \quad 0.9^2(20), \quad 0.9^3(20), \dots$$

| 1st swing | 2nd swing | 3rd swing | 4th swing |

After 10 swings, what is the total length of the distance the pendulum has swung?

76. A pendulum swings through an arc of 16 inches. On each successive swing, the length of the arc is 96% of the previous length.

$$16, \quad 0.96(16), \quad (0.96)^2(16), \quad (0.96)^3(16), \dots$$

| 1st swing | 2nd swing | 3rd swing | 4th swing |

After 10 swings, what is the total length of the distance the pendulum has swung?

Use the formula for the value of an annuity to solve Exercises 77–84. Round answers to the nearest dollar.

77. To save money for a sabbatical to earn a master's degree, you deposit $2000 at the end of each year in an annuity that pays 7.5% compounded annually.

    a. How much will you have saved at the end of five years?

    b. Find the interest.

78. To save money for a sabbatical to earn a master's degree, you deposit $2500 at the end of each year in an annuity that pays 6.25% compounded annually.

    a. How much will you have saved at the end of five years?

    b. Find the interest.

79. At age 25, to save for retirement, you decide to deposit $50 at the end of each month in an IRA that pays 5.5% compounded monthly.

    a. How much will you have from the IRA when you retire at age 65?

    b. Find the interest.