Exercise Set 11.1

Practice Exercises

In Exercises 1–12, write the first four terms of each sequence whose general term is given.

1. $a_n = 3n + 2$
2. $a_n = 4n - 1$
3. $a_n = 3^n$
4. $a_n = \left(\frac{1}{3}\right)^n$
5. $a_n = (-3)^n$
6. $a_n = \left(-\frac{1}{3}\right)^n$
7. $a_n = (-1)^n(n + 3)$
8. $a_n = (-1)^{n+1}(n + 4)$
9. $a_n = \frac{2n}{n + 4}$
10. $a_n = \frac{3n}{n + 5}$
11. $a_n = \frac{(-1)^{n+1}}{2^n - 1}$
12. $a_n = \frac{(-1)^{n+1}}{2^n + 1}$

The sequences in Exercises 13–18 are defined using recursion formulas. Write the first four terms of each sequence.

13. $a_1 = 7$ and $a_n = a_{n-1} + 5$ for $n \geq 2$
14. $a_1 = 12$ and $a_n = a_{n-1} + 4$ for $n \geq 2$
15. $a_1 = 3$ and $a_n = 4a_{n-1}$ for $n \geq 2$
16. $a_1 = 2$ and $a_n = 5a_{n-1}$ for $n \geq 2$
17. $a_1 = 4$ and $a_n = 2a_{n-1} + 3$ for $n \geq 2$
18. $a_1 = 5$ and $a_n = 3a_{n-1} - 1$ for $n \geq 2$

In Exercises 19–22, the general term of a sequence is given and involves a factorial. Write the first four terms of each sequence.

19. $a_n = \frac{n^2}{n!}$
20. $a_n = \frac{(n + 1)!}{n^2}$
21. $a_n = 2(n + 1)!$
22. $a_n = -2(n - 1)!$

In Exercises 23–28, evaluate each factorial expression.

23. $\frac{17!}{15!}$
24. $\frac{18!}{16!}$
25. $\frac{16!}{2!14!}$
26. $\frac{20!}{2!18!}$
27. $\frac{(n + 2)!}{n!}$
28. $\frac{(2n + 1)!}{(2n)!}$

In Exercises 29–42, find each indicated sum.

29. $\sum_{i=1}^{6} 5i$
30. $\sum_{i=1}^{6} 7i$
31. $\sum_{i=1}^{4} 2i^2$
32. $\sum_{i=1}^{5} i^3$
33. $\sum_{k=1}^{5} k(k + 4)$
34. $\sum_{k=1}^{4} (k - 3)(k + 2)$
35. $\sum_{i=1}^{4} \left(-\frac{1}{2}\right)^i$
36. $\sum_{i=2}^{4} \left(-\frac{1}{3}\right)^i$
37. $\sum_{i=5}^{9} 11$
38. $\sum_{i=3}^{7} 12$
39. $\sum_{i=0}^{4} \frac{(-1)^i}{i!}$
40. $\sum_{i=0}^{4} \frac{(-1)^{i+1}}{(i + 1)!}$
41. $\sum_{i=1}^{5} \frac{i!}{(i - 1)!}$
42. $\sum_{i=1}^{5} \frac{(i + 2)!}{i!}$

In Exercises 43–54, express each sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

43. $1^2 + 2^2 + 3^2 + \cdots + 15^2$
44. $1^4 + 2^4 + 3^4 + \cdots + 12^4$
45. $2 + 2^2 + 2^3 + \cdots + 2^{11}$
46. $5 + 5^2 + 5^3 + \cdots + 5^{12}$
47. $1 + 2 + 3 + \cdots + 30$
48. $1 + 2 + 3 + \cdots + 40$
49. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{14}{14 + 1}$
50. $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \cdots + \frac{16}{16 + 2}$
51. $4 + \frac{4^2}{2} + \frac{4^3}{3} + \cdots + \frac{4^n}{n}$
52. $\frac{1}{9} + \frac{2}{9^2} + \frac{3}{9^3} + \cdots + \frac{n}{9^n}$
53. $1 + 3 + 5 + \cdots + (2n - 1)$
54. $a + ar + ar^2 + \cdots + ar^{n-1}$

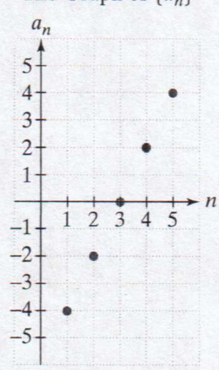
In Exercises 55–60, express each sum using summation notation. Use a lower limit of summation of your choice and k for the index of summation.

55. $5 + 7 + 9 + 11 + \cdots + 31$
56. $6 + 8 + 10 + 12 + \cdots + 32$
57. $a + ar + ar^2 + \cdots + ar^{12}$
58. $a + ar + ar^2 + \cdots + ar^{14}$
59. $a + (a + d) + (a + 2d) + \cdots + (a + nd)$
60. $(a + d) + (a + d^2) + \cdots + (a + d^n)$

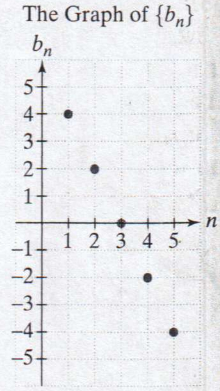
Practice Plus

In Exercises 61–68, use the graphs of $\{a_n\}$ and $\{b_n\}$ to find each indicated sum.

The Graph of $\{a_n\}$



The Graph of $\{b_n\}$



61. $\sum_{i=1}^{5} (a_i^2 + 1)$
62. $\sum_{i=1}^{5} (b_i^2 - 1)$
63. $\sum_{i=1}^{5} (2a_i + b_i)$
64. $\sum_{i=1}^{5} (a_i + 3b_i)$

65. $\sum_{i=4}^{5}\left(\frac{a_i}{b_i}\right)^2$

66. $\sum_{i=4}^{5}\left(\frac{a_i}{b_i}\right)^3$

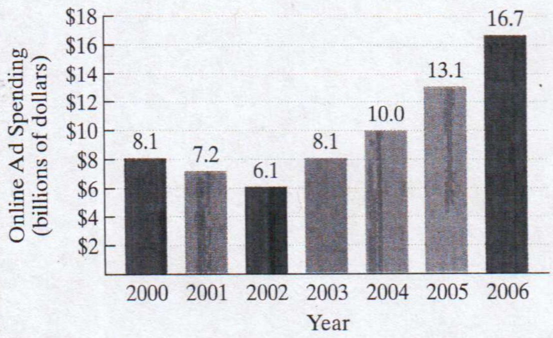
67. $\sum_{i=1}^{5} a_i^2 + \sum_{i=1}^{5} b_i^2$

68. $\sum_{i=1}^{5} a_i^2 - \sum_{i=3}^{5} b_i^2$

Application Exercises

69. Advertisers don't have to fear that they'll face a sea of "sold out" signs as they rush to the Internet. The growing number of popular sites filled with user-created content, including MySpace.com and YouTube.com, provide plenty of inventory for advertisers who can't find space on top portals such as Yahoo. The bar graph shows U.S. online ad spending, in billions of dollars, from 2000 through 2006.

United States Online Ad Spending



Source: eMarketer

Let a_n represent online ad spending, in billions of dollars, n years after 1999.

a. Use the numbers given in the graph to find and interpret $\frac{1}{7}\sum_{i=1}^{7} a_i$.

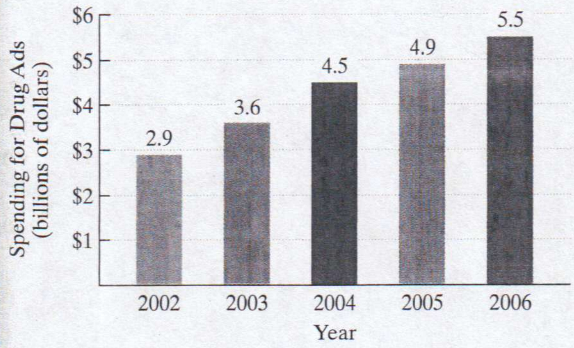
b. The finite sequence whose general term is

$$a_n = 0.5n^2 - 1.5n + 8,$$

where $n = 1, 2, 3, \ldots, 7$, models online ad spending, a_n, in billions of dollars, n years after 1999. Use the model to find $\frac{1}{7}\sum_{i=1}^{7} a_i$. Does this underestimate or overestimate the actual sum in part (a)? By how much?

70. More and more television commercial time is devoted to drug companies as hucksters for the benefits and risks of their wares. The bar graph shows the amount that drug companies spent on consumer drug ads, in billions of dollars, from 2002 through 2006.

Spending for Consumer Drug Ads



Source: Nielsen Monitor-Plus

Let a_n represent spending for consumer drug ads, in billions of dollars, n years after 2001.

a. Use the numbers given in the graph to find and interpret $\frac{1}{5}\sum_{i=1}^{5} a_i$.

b. The finite sequence whose general term is

$$a_n = 0.65n + 2.3,$$

where $n = 1, 2, 3, 4, 5$, models spending for consumer drug ads, in billions of dollars, n years after 2001. Use the model to find $\frac{1}{5}\sum_{i=1}^{5} a_i$. Does this seem reasonable in terms of the actual sum in part (a), or has model breakdown occurred?

71. A deposit of \$6000 is made in an account that earns 6% interest compounded quarterly. The balance in the account after n quarters is given by the sequence

$$a_n = 6000\left(1 + \frac{0.06}{4}\right)^n, \qquad n = 1, 2, 3, \ldots.$$

Find the balance in the account after five years. Round to the nearest cent.

72. A deposit of \$10,000 is made in an account that earns 8% interest compounded quarterly. The balance in the account after n quarters is given by the sequence

$$a_n = 10{,}000\left(1 + \frac{0.08}{4}\right)^n, \qquad n = 1, 2, 3, \ldots.$$

Find the balance in the account after six years. Round to the nearest cent.

Writing in Mathematics

73. What is a sequence? Give an example with your description.

74. Explain how to write terms of a sequence if the formula for the general term is given.

75. What does the graph of a sequence look like? How is it obtained?

76. What is a recursion formula?

77. Explain how to find $n!$ if n is a positive integer.

78. Explain the best way to evaluate $\frac{900!}{899!}$ without a calculator.

79. What is the meaning of the symbol Σ? Give an example with your description.

80. You buy a new car for \$24,000. At the end of n years, the value of your car is given by the sequence

$$a_n = 24{,}000\left(\frac{3}{4}\right)^n, \qquad n = 1, 2, 3, \ldots.$$

Find a_5 and write a sentence explaining what this value represents. Describe the nth term of the sequence in terms of the value of your car at the end of each year.

Technology Exercises

In Exercises 81–85, use a calculator's factorial key to evaluate each expression.

81. $\frac{200!}{198!}$

82. $\left(\frac{300}{20}\right)!$

83. $\frac{20!}{300}$