

Algebra 2

Unit 1 Lesson 7: Logarithms and Exponentials

- Introduce a Logarithm
 - Converting between Exponential and Logarithmic forms
 - Evaluating basic logarithms
-

Solve these for x by testing values.

1. $2^x = 32$
5

2. $2^x = 8$ 3

The problem we face... $2^x = 6$

So we know $2^5 = 32$ and $2^3 = 8$... But 2 raised to what power gives 6?

Try some guesses in your calculator and come up with an estimate.

$$2^{2.5} = 5.66$$

$$2^{2.75} = 6.73$$

$$2^{2.6} = 6.06$$

$$2^{2.59} = 6.02$$

$$2^{2.58} = 5.98$$

Logarithms were defined to answer these types of problems.

Definition of Logarithm with Base b:

$$b^x = y \quad \text{iff} \quad \log_b y = x$$

Convert the previous example to a logarithm equation to solve for x.

$$2^x = 6$$
$$\log_2 6 = x$$

Memorize This : Logarithms are used to solve for exponents!

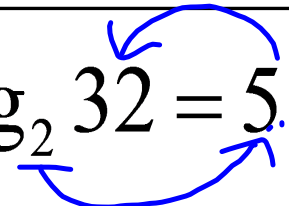
Restrictions.

- Base must be positive. ✱
- Can only take the log of a positive numbers.

Examples: Converting To Exponential Form

$$\log_b y = x \quad \text{iff} \quad b^x = y$$

Logarithmic Form

$$\log_2 32 = 5$$


$$\log_5 1 = 0$$

$$\log_{10} 10 = 1$$

$$\log_{\frac{1}{2}} 2 = -1$$

Exponential Form

$$2^5 = 32$$

$$5^0 = 1$$

$$10^1 = 10$$

$$\frac{1}{2}^{-1} = 2$$

Examples: Converting To Exponential Form

$$\log_b y = x \quad \text{iff} \quad b^x = y$$

Exponential Form

$$2^8 = 256$$

$$3^x = 5$$

$$a^x = b$$

$$8^{2x-1} = 510$$

$$3^{\log_3 x} = x$$

Logarithmic Form

$$\log_2 256 = 8$$

$$\log_3 5 = x$$

$$\log_a b = x$$

$$\log_8 510 = 2x-1$$

$$\log_3 9 = \log_3 x$$

Evaluating logarithms by hand:

Ask yourself, "The base raised to what power gives me the number inside the log?"

Evaluate $\log_2 32$ $2^x = 32$

Ask yourself : Two raised to what power is 32?

5

Ex: Evaluate $\log_4 16 = 2$

Ex: Evaluate $\log_3 1 = 0$ ~~4~~

"Common Log" and "Natural Log":

Common Log is when logarithmic expressions don't show a base. The base is understood to be 10. (The log button on your calculator is actually common log)

Natural Log is a logarithm with the base of e. It has a separate abbreviation and looks like this.

$$\log_{10} 100 = \log 100 = 2$$

$$\log_e 1 = \ln 1 = 0$$

Special Logarithm Values:

$\log_b 1 = 0$ since $b^0 = 1$

$\log_b b = 1$ since $b^1 = b$

$\log_7 7 = 1$

Examples:

$\log_{10} 10 = 1$

$\ln 1 = 0$

$\log_7 1 = 0$

$\ln e = 1$
 $\log_e e = 1$

Homework:

Worksheet called "Algebra 2: Converting and Evaluating Logs"