Algebra 2

Unit 1 Lesson 7: Logarithms and Exponentials

- Introduce a Logarithm
- Converting between Exponential and Logarithmic forms
- Evaluating basic logarithms

Solve these for x by testing values.

1. $2^x = 32$ 5

2. $2^x = 8$ 3

The problem we face... $2^x = 6$

So we know $2^5 = 32$ and $2^3 = 8$... But 2 raised to what power gives 6?

Try some guesses in your calculator and come up with an estimate.

$2^{2.5} = 5.66$

$2^{2.75} = 6.73$

$2^{2.6} = 6.06$

$2^{2.59} = 6.02$

$2^{2.58} = 5.98$

Logarithms were defined to answer these types of problems.

Definition of Logarithm with Base b:

$b^x = y$ iff $\log_b y = x$

Convert the previous example to a logarithm equation to solve for x.

$2^x = 6$

$\log_2 6 = x$

Memorize This : Logarithms are used to solve for exponents!

Restrictions.

- Base must be positive.
- Can only take the log of a positive numbers.

Examples: Converting To Exponential Form

$\log_b y = x$ iff $b^x = y$

Logarithmic Form	Exponential Form
$\log_2 32 = 5$	$2^5 = 32$
$\log_5 1 = 0$	$5^0 = 1$
$\log_{10} 10 = 1$	$10^1 = 10$
$\log_{\frac{1}{2}} 2 = -1$	$\left(\frac{1}{2}\right)^{-1} = 2$

Examples: Converting To Exponential Form

$\log_b y = x$ iff $b^x = y$

Exponential Form	Logarithmic Form
$2^8 = 256$	$\log_2 256 = 8$
$3^x = 5$	$\log_3 5 = x$
$a^x = b$	$\log_a b = x$
$8^{2x-1} = 510$	$\log_8 510 = 2x-1$
$3^{\log_3 x} = 9$	$\log_3 9 = \log_3 x$

Evaluating logarithms by hand:

Ask yourself, "The base raised to what power gives me the number inside the log?"

Evaluate $\log_2 32$ $2^x = 32$

Ask yourself : Two raised to what power is 32?

5

Ex: Evaluate $\log_4 16 =$ 2

Ex: Evaluate $\log_3 1 =$ 0

"Common Log" and "Natural Log":

Common Log is when logarithmic expressions don't show a base. The base is understood to be 10. (The log button on your calculator is actually common log)

Natural Log is a logarithm with the base of *e*. It has a separate abbreviation and looks like this.

$\log_{10} 100 = \log 100 =$ 2

$\log_e 1 = \ln 1 =$ 0

Special Logarithm Values:

$\log_b 1 = 0$ since $b^0 = 1$

$\log_b b = 1$ since $b^1 = b$

$\log_7 7 = 1$

Examples:

$\log 10 = 1$ ($\log_{10}$)

$\ln 1 = 0$

$\log_7 1 = 0$

$\ln e = 1$ ($\log_e e$)

Homework:

Worksheet called "Algebra 2: Converting and Evaluating Logs"