

Algebra 2

Chapter 11 Review - Pt. 1

Name: Key

Hour: _____

State whether the following sequences are arithmetic, geometric, or neither. Then state the common difference or ratio (if it exists)

1. $3.5, 4.75, 6, 7.25, \dots$ $d = 1.25$

Answer: Arithmetic $d = 1.25$

2. $1009, 996, 982, 967, \dots$

Answer: Neither

3. $18, 14.4, 11.52, 9.216, \dots$ $\frac{14.4}{18} = 0.8, \frac{11.52}{14.4} = 0.8$

Answer: Geometric $r = 0.8$

4. $7, -14, 28, -56, \dots$ $\frac{28}{-14} = -2$

Answer: Geometric $r = -2$

5. $25, 31, 37, 44, \dots$

Answer: Neither

6. $-22, -15, -8, -1, \dots$

Answer: Arithmetic $d = 7$

Formulas needed: $a_n = a_1 + (n-1) \cdot d$ and $S_n = \frac{n}{2}(a_1 + a_n)$

For the following arithmetic sequences, do the following: Write the general rule, find the 15th term, then sum the first 15 terms.

7. $-5, -1, 3, 7, \dots$ $a_n = -5 + (n-1) \cdot 4$
 $a_{15} = -5 + (15-1) \cdot 4 = 51$
 $S_{15} = \frac{15}{2}(-5 + 51)$

General Rule: $a_n = -5 + (n-1) \cdot 4$

15th term: 51

Sum of first 15 terms: 345

8. $37, 34, 31, 28, \dots$ $a_n = 37 + (n-1) \cdot (-3)$
 $a_{15} = 37 + (15-1) \cdot (-3) = -5$
 $S_{15} = \frac{15}{2}(37 + (-5)) = 240$

General Rule: $a_n = 37 + (n-1) \cdot (-3)$

15th term: -5

Sum of first 15 terms: 240

Find a_1 : $9, \frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{11}{6}, \dots$
 $\frac{1}{3} + d = \frac{5}{6}$ $d = \frac{5}{6} - \frac{1}{3}$ $a_n = \frac{1}{3} + (n-1) \cdot \frac{1}{2}$
 $- \frac{1}{3}$ $d = \frac{3}{6} = \frac{1}{2}$ $a_{15} = \frac{1}{3} + (15-1) \cdot \frac{1}{2} =$
 $S_{15} = \frac{15}{2}\left(\frac{1}{3} + \frac{22}{3}\right)$

General Rule: $a_n = \frac{1}{3} + (n-1) \cdot \frac{1}{2}$

15th term: $\frac{22}{3}$

Sum of first 15 terms: 57.5

10. $1, 1.3, 1.6, 1.9, \dots$ $a_n = 1 + (n-1) \cdot 0.3$
 $a_{15} = 1 + (15-1) \cdot 0.3$
 $= 5.2$

General Rule: $a_n = 1 + (n-1) \cdot 0.3$

15th term: 5.2

Sum of first 15 terms: 46.5

$S_{15} = \frac{15}{2}(1 + 5.2)$
 $= 46.5$

Formulas needed:

$$a_n = a_1 \cdot r^{n-1}$$

and

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

For the following geometric sequences, do the following: write the general rule, state the 11th term, and sum the first 9 terms.

11. $2, -8, 32, -128, \dots$

$$\begin{array}{r} 1 \\ -4 \\ \times 4 \end{array}$$

$$r = -4$$

$$a_n = 2 \cdot (-4)^{n-1}$$

$$a_{11} = 2(-4)^{10-1} =$$

$$S_9 = \frac{2(1 - (-4)^9)}{1 - (-4)} =$$

General Rule: $a_n = 2(-4)^{n-1}$

11th term: 2097152

Sum of first 9 terms: 104858

12. $100, 50, 25, 12.5, \dots$

$$\begin{array}{r} 1 \\ \frac{1}{2} \\ \times \frac{1}{2} \\ \frac{1}{2} \end{array}$$

$$r = \frac{1}{2}$$

$$a_n = 100\left(\frac{1}{2}\right)^{n-1}$$

$$a_{11} = 100\left(\frac{1}{2}\right)^{10} =$$

$$S_9 = \frac{100(1 - (\frac{1}{2})^9)}{1 - (\frac{1}{2})} =$$

General Rule: $a_n = 100\left(\frac{1}{2}\right)^{n-1}$

11th term: 0.098 rounded

Sum of first 9 terms: 199.6 rounded

13. $9, 12, 16, \frac{64}{3}, \dots$

$$\begin{array}{r} 1 \\ \frac{4}{3} \\ \times \frac{4}{3} \\ \frac{4}{3} \end{array}$$

$$a_n = 9\left(\frac{4}{3}\right)^{n-1}$$

$$a_{11} = 9\left(\frac{4}{3}\right)^{10-1} = 159.8$$

$$S_9 = \frac{9(1 - (\frac{4}{3})^9)}{1 - \frac{4}{3}} = 332.6$$

General Rule: $a_n = 9\left(\frac{4}{3}\right)^{n-1}$

11th term: 159.8

Sum of first 9 terms: 332.6

14. $2.5, -5, 10, -20, \dots$

$$\begin{array}{r} 1 \\ -2 \\ \times -2 \\ -2 \end{array}$$

$$r = -2$$

$$a_{11} = 2.5(-2)^{10-1} =$$

$$S_9 = \frac{2.5(1 - (-2)^9)}{1 - (-2)} =$$

General Rule: $a_n = 2.5(-2)^{n-1}$

11th term: 2560

Sum of first 9 terms: 427.5

Evaluate the problems given in Summation Notation.

15. $\sum_{n=3}^6 3(-2)^{n-1}$

$$3(-2)^2 + 3(-2)^3 + 3(-2)^4 + 3(-2)^5 = -60$$

$$3(4) + 3(-8) + 3(16) + 3(-32)$$

16. $\sum_{k=1}^4 3(k-1)$

$$3(1-1) + 3(2-1) + 3(3-1) + 3(4-1)$$

$$3(0) + 3(1) + 3(2) + 3(3)$$

$$0 + 3 + 6 + 9$$

$$= 18$$

17. $\sum_{n=10}^{13} -n(n-5)$

$$-10(10-5) + -11(11-5) + -12(12-5) + -13(13-5)$$

$$-10(5) + -11(6) + -12(7) + -13(8)$$

$$= -304$$

Algebra 2

Chapter 11 Test Review – Part 2

Name: Key

Hour: _____

State whether the following geometric sequences will converge or diverge. Then state what $|r|$ equals. You do not have to find the sum of the converging sequences in this section.

1. $a_n = 5(-0.5)^{n-1}$

Converge / Diverge, because $|r| = \underline{0.5}$

2. $a_n = \pi \left(\frac{7}{6}\right)^{n-1}$

Converge / Diverge, because $|r| = \underline{\frac{7}{6}}$

3. 15, 11.25, 8.4375, ...

$$\begin{array}{l} 11.25 \div 15 \\ \hline r = 0.75 \end{array}$$

Converge / Diverge, because $|r| = \underline{0.75}$

4. $\sum_{n=1}^{\infty} -4 \left(-\frac{11}{12}\right)^{n-1}$

Converge / Diverge, because $|r| = \underline{\frac{11}{12}}$

5. 2, -4, 8, -16, ...

$$\begin{array}{l} -528 \div 1056 \\ \hline r = -2 \end{array}$$

Converge / Diverge, because $|r| = \underline{2}$

6. 1056, -528, 264, -132, ...

$$\begin{array}{l} -528 \div 1056 \\ \hline r = -0.5 \end{array}$$

Converge / Diverge, because $|r| = \underline{0.5}$

7. $\sum_{n=1}^{\infty} 10(1.25)^{n-1}$

Converge / Diverge, because $|r| = \underline{1.25}$

Find the sum of the following infinite geometric sequences.

8. $\sum_{n=1}^{\infty} 7 \left(-\frac{5}{6}\right)^{n-1}$

$$a_1 = 7 \quad r = -\frac{5}{6}$$

$$\begin{aligned} S &= \frac{7}{1 - \left(-\frac{5}{6}\right)} = \frac{7}{\frac{11}{6}} \\ &= \boxed{\frac{42}{11} \approx 3.82} \end{aligned}$$

Formula:
 $S = \frac{a_1}{1-r}$ {if $|r| < 1$ }

9. $\sum_{n=1}^{\infty} 8(0.93)^{n-1}$

$$a_1 = 8 \quad r = 0.93$$

$$S = \frac{8}{1 - (0.93)} = \frac{8}{0.07} = \boxed{\frac{800}{7} \approx 114.3}$$

10. $\frac{11}{2}, \frac{11}{4}, \frac{11}{8}, \dots$

$$a_1 = \frac{11}{2} \quad r = \frac{11}{4} \div \frac{11}{2} = \frac{1}{2} \quad S = \frac{\frac{11}{2}}{1 - \frac{1}{2}} = \frac{\frac{11}{2}}{\frac{1}{2}} = \boxed{11}$$

11. $\frac{7}{9}, \frac{7}{36}, \frac{7}{144}, \dots$

$$a_1 = \frac{7}{9} \quad r = \frac{7}{36} \div \frac{7}{9} = 0.25 \quad S = \frac{\frac{7}{9}}{1 - 0.25} = \frac{\frac{7}{9}}{0.75} = \frac{7}{9} \cdot \frac{4}{3} = \boxed{\frac{28}{27} \approx 1.04}$$

Write a recursive rule for the following sequences.

12. 4, 9, 14, 19, ...

$$a_1 = 4 \quad a_n = a_{n-1} + 5$$

13. $1, -\frac{1}{4}, \frac{1}{16}, -\frac{1}{64}, \dots$

$$a_1 = 1, \quad a_n = a_{n-1} \cdot (-0.25)$$

14. 1000, 993, 986, 979, ... ← Arithmetic
 $d = -7$

$a_1 = 1000, a_n = a_{n-1} - 7$

15. $\frac{3}{5}, \frac{9}{10}, \frac{27}{20}, \dots$ ← Geometric $r = 1.5$

$a_1 = \frac{3}{5}, a_n = a_{n-1} \cdot 1.5$

In your own words, what is an arithmetic sequence?

A sequence where consecutive terms are found by taking the previous term and adding the same number. Ex: 1, 4, 7, 10, ...
 $\downarrow \downarrow \downarrow$
 $+3 +3 +3$

In your own words, what is a geometric sequence?

A sequence where every term divided by its previous term (excluding the first term) will give you the same value.

What is the notation for the 15th term in a sequence? What about the 250th term?

$\underline{a_{15}}$

$\underline{a_{250}}$

Find the sum of the first 16 terms of the following sequences:

16. 4, 7, 10, 13, ...

Arithmetic: $a_1 = 4, d = 3$ General rule: $a_n = 4 + (n-1) \cdot 3$

Sum: $S_{16} = \frac{16}{2} \left(4 + \underbrace{49}_{\uparrow} \right) = \boxed{424}$
 a_{16} goes here

$$\begin{aligned} a_{16} &= 4 + (16-1) \cdot 3 \\ &= 4 + 15 \cdot 3 \\ &= 49 \end{aligned}$$

17. 10, -5, 2.5, -1.25, ...

Geometric: $a_1 = 10, r = -\frac{1}{2}$

Sum: $S = \frac{10 \left(1 - \left(-\frac{1}{2} \right)^{16} \right)}{1 - \left(-\frac{1}{2} \right)}$ $\approx \boxed{6.67}$

Formula $S = \frac{a_1 (1 - r^n)}{1 - r}$